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# The Theory of Giant Splash of Photoresponse in Semiconductors at Low-Level Illumination with Increasing Concentration of Deep Recombination Impurity

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Additional information is available at the end of the chapter

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## Abstract

Recombination of excess (nonequilibrium) electrons and holes in semiconductors through impurity recombination centers (traps) known as trap-assisted (Shockley-Read-Hall) recombination is in many cases the dominant process. In this chapter, we develop the general theory of trap-assisted recombination and study in detail two key characteristics: (1) dependences of excess charge carriers' lifetime and photoelectric gain on concentration  $N$  of recombination centers and (2) effectiveness of band-to-band photoexcitation of charge carriers and photo-emf in semiconductors at low-level illumination considered outside quasi-neutrality approximation.

We have done systematic mathematical and detailed physical analysis of considered characteristics. Giant splash of photoresponse in semiconductors with increasing recombination center concentration  $N$  is caused mainly by the growth of charge carriers' lifetime in orders of magnitude. Also, this factor is sufficient to provide an increase, in order of magnitude and more, in efficiency of charge carriers' photoexcitation with increasing  $N$ . Results of strict analytical calculations (i.e., outside commonly used local approximation of quasi-neutrality) show, that, photoinduced local space charge affects substantially on giant splash of semiconductor photoelectric response with increasing concentration of recombination centers.

The theory of giant splash of photoresponse in semiconductors at low-level illumination with increasing concentration of recombination centers could develop further through generalization of boundary conditions on semiconductor surfaces and current contact electrodes, accounting for nonuniformity of charge carriers' photoexcitation along the line of current flow and fluctuation processes. The study of nonstationary (frequency domain and transient) characteristics is of particular interest.

It follows from physical essence of considered effects that similar effects can occur in other mediums with recombination of dissociative or ion-ion type, for example, in multicomponent plasma as well.

**Keywords:** Impurity recombination center (trap), Trap-assisted (Shockley-Read-Hall) recombination, Excess (nonequilibrium) charge carriers, Band-to-band photogeneration of excess charge carriers, Electron lifetime, Hole lifetime, Photoresponse, Photoelectric gain, Photo-emf, Small deviation from equilibrium state in semiconductor, Photoinduced space charge, Approximation of quasi-neutrality, Giant splash of photoresponse

## 1. Introduction

Recombination of excess (nonequilibrium) electrons and holes in semiconductors through impurity recombination centers (traps) referred to in this chapter as trap-assisted (Shockley-Read-Hall) recombination is in many cases the dominant process [1-9]. Hall [10] and Shockley and Read [11] have proposed the theory of trap-assisted recombination as early as 1952. Further, the theory of trap-assisted recombination has been developed in many aspects and details [1-5, 12-14]. At the same time, due attention was not paid to study dependences of lifetimes of excess electrons  $\tau_n(N)$  and holes  $\tau_p(N)$  on concentration of traps  $N$ . In some cases, traps are produced intentionally by doping semiconductor (e.g., by bombarding with high-energy ions [15, 16]) to reduce time of transient processes. It seems that lack of attention is caused by traditional understanding that the larger the concentration of traps  $N$ , the greater the capture rate of excess charge carriers on impurity level traps and, therefore, the shorter the lifetimes of excess charge carriers.

That reasonable understanding is incompletely adequate to reality. As shown below, lifetimes of excess electrons and holes (see our definition of  $\tau_n$  and  $\tau_p$  in Section 2) may grow strongly (in order of magnitude and more) with increase of concentration  $N$ .

In this chapter, we generalize the theory and give systematic mathematical and detailed physical analysis of dependences  $\tau_n(N)$  and  $\tau_p(N)$  on concentration of recombination centers.

## 2. Preliminaries

Let's consider the case of small deviation of free charge carriers' concentrations from equilibrium values. This situation occurs often in semiconductors used for registration of low-level signals, for example, optical signals. It will be shown that both  $\tau_n(N)$  and  $\tau_p(N)$ , under certain conditions, will grow very strongly with increasing  $N$  in a particular interval of  $N$  values. Completely different physical mechanism causes this increase in lifetimes. It differs from mechanisms available for many years [4], as well as proposed in [17]. We analyze extreme points (corresponding formulas are derived) of dependences  $\tau_n(N)$  and  $\tau_p(N)$  as functions of

semiconductor parameters and temperature. We give detailed physical interpretation of obtained results. In particular, it is shown that physical mechanisms responsible for strong non-monotonic dependences  $\tau_n(N)$  and  $\tau_p(N)$  differ from each other.

It is reasonable to expect that the growth of lifetimes  $\tau_n(N)$  and  $\tau_p(N)$  with increasing  $N$  will lead to the growth of photoresponse of semiconductor sample (including photoelectric gain  $G$ ). However, specificity of dependences  $\tau_n(N)$  and  $\tau_p(N)$  does not determine the type of dependence  $G(N)$  in total. As it follows from [18, 19],  $G$  increases with increasing charge carriers' lifetime, if ambipolar mobility [2, 13]  $\mu_a=0$  or there is no recombination at current contact electrodes ( $x=0$  and  $x=W$ ; see insert in Figure 1a). In reality, recombination at current contact electrodes is always happening to some extent [5-9]. Therefore, in normal conditions ( $\mu_a \neq 0$ ), increase in lifetimes, after reaching some values, does not lead to an increase in photocurrent  $I_{ph}$  [5, 9, 18, 19]. Saturation of  $I_{ph}$  becomes apparent in the case of high-rate recombination at the contact electrodes (sweep-out effect [5, 9, 18, 19]) when

$$\Delta n(0) = \Delta p(0) = \Delta n(W) = \Delta p(W) = 0, \quad (1)$$

where  $\Delta n(x) = n(x) - n_e$  and  $\Delta p(x) = p(x) - p_e$  are deviations of electron  $n$  and hole  $p$  concentrations from their equilibrium values  $n_e$  and  $p_e$ , respectively. In trap-assisted recombination, function  $\mu(N)$  can vanish at the same (up to small corrections) concentration  $N$ , at which dependences  $\tau_n(N)$  and  $\tau_p(N)$  reach points of maximal extremum  $\hat{\tau}_n$  and  $\hat{\tau}_p$  (Figure 1b and 1c). Therefore  $G$  and, hence,  $I_{ph}$  grow to the extent of increase in  $\tau_n(N)$  and  $\tau_p(N)$ . These are physical grounds of giant splash of photoelectric gain with increasing  $N$  (Figure 1a).

It was first reported in [20] that vanishing  $\mu(N)$  in points of maximal extremum of dependences  $\tau_n(N)$  and  $\tau_p(N)$  allows avoiding highly undesirable effect – saturation of  $G$  in intrinsic photoconductors, when applied bias voltage  $V$  increases [5, 9, 18, 19, 21, 22]. As is known [19], this disadvantage is the most evident in photoconductors with sweep-out effect on contact electrodes, i.e., when relations (1) are fulfilled. Result presented in Ref. [20] was obtained in approximation of quasi-neutrality [1-9, 13, 18, 19, 21, 22], which was usually used at moderate electric fields, i.e., when we neglect in Poisson equation by term  $\Delta \rho \equiv (\varepsilon \times \varepsilon_0) \times \text{div} \Delta E$ , which determines the density of photoinduced space charge  $\Delta \rho$ .

In our case,  $\Delta E \equiv E(x) - E_0$  is the variation of electric field caused by deviation of concentrations of free charge carriers and their traps from equilibrium values by reason of band-to-band absorption of radiation,  $E(x)$  and  $E_0$  are the electric field intensity in the presence and absence of illumination,  $\varepsilon$  is the relative dielectric permittivity of semiconductor, and  $\varepsilon_0$  is the vacuum permittivity. However, even at moderate electric fields ( $\approx 1\div 10$  V/cm), approximation of quasi-neutrality is not always acceptable [23].

Below, in case of single recombination level, we consider in detail the impact of photoinduced space charge  $\Delta \rho$  on value  $G$  of photoelectric gain  $G$  in semiconductors with sweep-out effect on contact electrodes at the point of maximal extremum of function  $G(N)$  (Figure 1a). Considering semiconductor as base material for making intrinsic photoconductors with threshold electro-optical performance, we assume that photocarriers are excited by photons of low-intensity optical radiation with wavelength range responding to fundamental absorption band of semiconductor. Figure 2a shows that we cannot use approximation of quasi-neutrality, when voltage  $V$  across the sample becomes larger than some particular value.

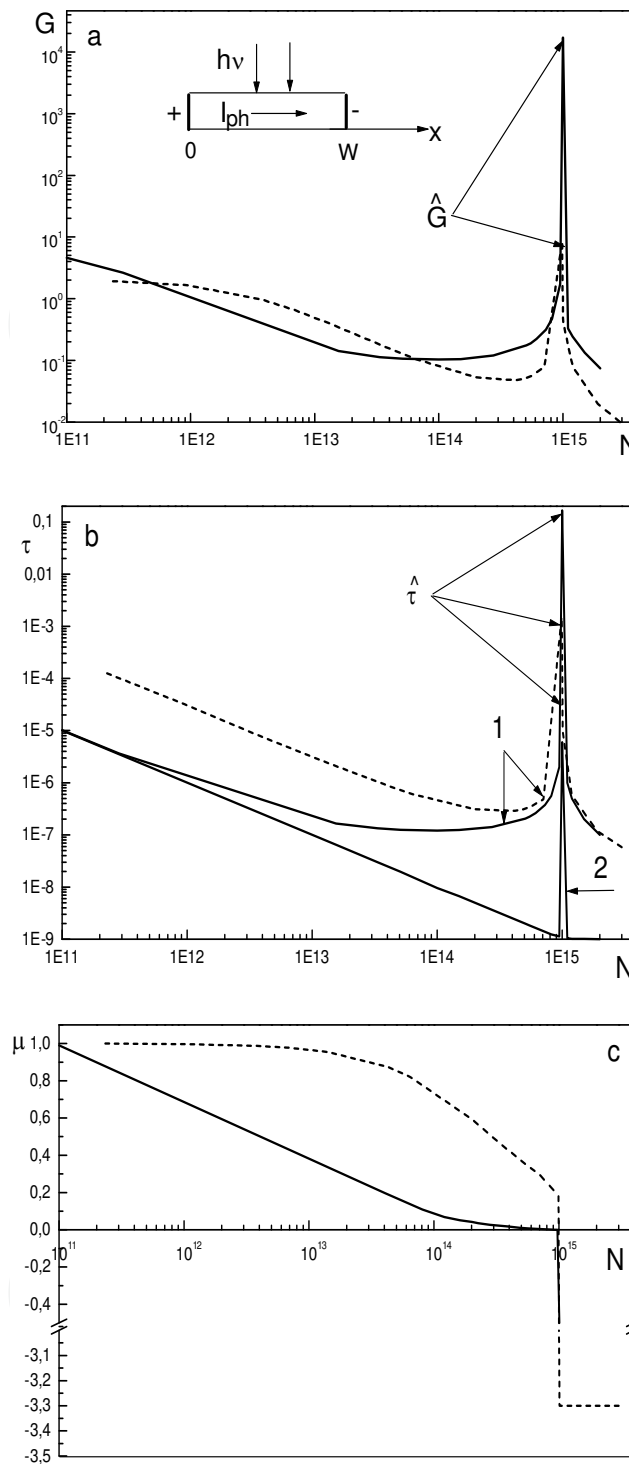
Also, ignoring approximation of quasi-neutrality, we study, at low-level illumination, the effectiveness of band-to-band photoexcitation of charge carriers and photo-emf  $V_{ph}$  in semiconductors with dominant trap-assisted recombination. Analytical expressions for photo-emf  $V_{ph}$  and mean, with respect to light propagation length, concentrations of photoelectrons  $\langle \Delta n \rangle$  and photoholes  $\langle \Delta p \rangle$  are given. It is shown that target values of  $V_{ph}$ ,  $\langle \Delta n \rangle$ , and  $\langle \Delta p \rangle$  can be improved radically by increasing concentration of recombination centers; moreover, approximation of quasi-neutrality can lead to errors of several orders of magnitude.

Analyzing above-mentioned problems, we do not use conventional (Shockley-Read) expression-based form [1-5, 9, 11-14] of generation-recombination rate. This form does not allow to express explicitly dependences  $\tau_n$  and  $\tau_p$  and, even more,  $I_{ph}$  and  $V_{ph}$  on  $N$ . And therefore, because of the need for solving complex transcendental equations, conventional (Shockley-Read) expression-based form leads to serious mathematical difficulties in study (especially analytical) dependences  $\tau_{n,p}(N)$  and, even more,  $I_{ph}(N)$  and  $V_{ph}(N)$ .

These difficulties are dramatized by the fact that under certain conditions,  $\tau_{n,p}$ ,  $I_{ph}$  and  $V_{ph}$  are dependent very strongly on concentration  $N$  in a particular interval of  $N$  (Figure 1a, 1b, 7). Perhaps, it was the main reason for longtime absence of complete theoretical analysis of lifetime dependences  $\tau_{n,p}(N)$ , while detailed analysis of lifetime dependences on concentrations of free charge carriers was made in pioneering paper by Shockley and Read [11] concerning the theory of recombination through impurity level trap. In present chapter, we use the method of calculation assuming that  $N$  is expressed in terms of the ratio of the number of filled recombination level states to the number of empty. This allows to avoid transcendental equations, in other words, to avoid the need to solve inverse problem. As a result, the described above approach simplifies greatly the mathematical analysis and physical interpretation of calculations of desired parameters.

### 3. Model and basic relations

Consider nondegenerated semiconductor doped by shallow fully ionized single type impurity (for definition donors) with concentration  $N_D$ . Recombination of excess charge carriers occurs in said semiconductor through the energy level of acceptor impurity atoms with concentration  $N$ , which can be in two charge states (assume in neutral and singly negatively charged). Concentration of recombination impurity atoms in neutral state corresponds to concentration of acceptor atoms  $N_0$ , which are simultaneously centers of electron capture and centers of thermal emission of holes. Concentration of recombination impurity atoms in negatively charged state corresponds to concentration of atoms  $N_- = N - N_0$ , which are capture centers of holes and, at the same time, centers of thermal emission of electrons. Described above is recombination through single-level trap [10-13] (Figure 5b), which is often dominant [1-5, 9, 14] and called Shockley-Read-Hall recombination.



**Figure 1.** Dependences on concentration of single-level recombination centers  $N$  ( $\text{cm}^{-3}$ ): (a)  $G$ – photoelectric gain; (b)  $\tau$ – lifetime of electrons (curve 1) and holes (curve 2) (s); (c)  $\mu$ – ambipolar mobility of charge carriers (in units of hole mobility). Adopted: room temperature,  $W=10^{-1}\text{cm}$ ,  $\theta \approx w_p/w_n=10^2$ ,  $w_n=10^{-8}\text{cm}^3/\text{s}$ ,  $n_i/n_t=10^4$ ,  $N_D=10^{15}\text{cm}^{-3}$ ,  $E_0=10\text{V/cm}$ . Solid curves,  $GaAs$ , and dashed curves,  $Si$ . Physical parameters of semiconductors are obtained from monograph [3]. Schematic view of photoconductor on insert in Figure 1a

Let either band-to-band excitation (Figure 5b) or injection on the contacts produce excess electrons and holes. Then, in stationary case, equation

$$R_n = R_p \quad (2)$$

determines the charge state of recombination impurity atoms.

Recombination-generation rates of electrons  $R_n$  and holes  $R_p$  due to capture of charge carriers by acceptor impurity traps and their thermal emission from recombination level states into permitted conduction or valence bands are equal to

$$R_n = (n \times N_0 - \delta^{-1} \times n_e \times N_-) \times w_n, R_p = (p \times N_- - \delta \times p_e \times N_0) \times w_p. \quad (3)$$

Here,  $w_n$  and  $w_p$  are electron and hole capture probabilities, respectively, at appropriate recombination level state,  $\delta = N_-^e / N_0^e$  (superscript indicates equilibrium values of concentration of recombination impurity atoms  $N$  in relevant charge states).

For small deviation of charge carriers' and their capture centers' concentrations  $\Delta N_0 = N_0 - N_0^e = -\Delta N_- = N_-^e - N_-$  from equilibrium values, we can linearize relations (2) and (3) with respect to proper deviations. Then taking into account Poisson equation

$$\Delta \rho \equiv \frac{\varepsilon}{4\pi} \times \frac{\partial \Delta E}{\partial x} = q \times [\Delta p - \Delta n - \Delta N_-], \quad (4)$$

we obtain

$$R_n = \frac{\Delta n}{\tau_n} + a_n \times \text{div} \Delta E, \quad (5)$$

$$R_p = \frac{\Delta p}{\tau_p} - a_p \times \text{div} \Delta E, \quad (6)$$

$$\Delta p = \frac{\tau_p}{\tau_n} \times \Delta n + (a_n + a_p) \times \tau_p \times \frac{d\Delta E}{dx}, \quad (7)$$

where

$$\frac{1}{\tau_n} = w_n \times N \times \frac{\delta \times \theta}{1 + \delta} \times \frac{N + (1 + \delta) \times (1 + \delta^{-1}) \times (n_e + p_e)}{\delta \times \theta \times N + (1 + \delta) \times (1 + \delta^{-1}) \times (n_e + \delta \times \theta \times p_e)}, \quad (8)$$

$$\frac{1}{\tau_p} = w_p \times N \times \frac{\delta}{1 + \delta} \times \frac{\delta \times N + (1 + \delta)^2 \times (n_e + p_e)}{\delta \times N + (1 + \delta^2) \times (n_e + \delta \times \theta \times p_e)}, \quad (9)$$



$$a_n = \frac{\varepsilon}{4\pi \times q} \times w_n \times N \times n_e \times \frac{\theta \times (1 + \delta)}{\delta \times \theta \times N + (1 + \delta) \times (1 + \delta^{-1}) \times (n_e + \delta \times \theta \times p_e)}, \quad (10)$$

$$a_p = \frac{\varepsilon}{4\pi \times q} \times w_p \times N \times p_e \times \frac{1 + \delta}{N + (1 + \delta) \times (1 + \delta^{-1}) \times (n_e + \delta \times \theta \times p_e)}, \quad (11)$$

$\Delta E$  is change in electric field caused by deviation of charge carriers' and capture centers' concentrations from equilibrium values, and  $q$  is absolute electron charge value and  $\theta = w_p / w_n$ . First terms in (5) and (6) mean recombination rates of excess electrons and holes (and therefore, symbols  $\tau_n$  and  $\tau_p$  mean their lifetimes) in quasi-neutrality with respect to electric field  $\Delta E$ , i.e., at sufficiently small values  $\text{div} \Delta E$  [1-3, 5, 9-14, 18, 24, 25]. We will use the same terminology for  $\tau_n$  and  $\tau_p$  in the case of failure to comply quasi-neutrality (see below); therefore values  $\tau_n$  and  $\tau_p$  will not depend on value  $\text{div} \Delta E$  in the present study.

High-performance photoconductors operate with extremely low-level illumination. Therefore, linear for  $g$  approximation, usually used in the theory of high-performance photodetectors [5-7, 9, 21, 22, 26], is correct in calculation of photoelectric gain  $G$ , where  $g$  is density of charge carriers' photoexcitation rate.

In view of the above provision, we can write expressions for the density of photocurrent components as follows:

$$\Delta I_n = q \times \mu_n \times (E_0 \times \Delta n + n_e \times \Delta E) + q \times D_n \times \frac{\partial \Delta n}{\partial x}, \quad (12)$$

$$\Delta I_p = q \times \mu_p \times (E_0 \times \Delta p + p_e \times \Delta E) - q \times D_p \times \frac{\partial \Delta p}{\partial x}. \quad (13)$$

where  $\mu_n$  and  $\mu_p$  are electron and hole mobility and  $D_n$  and  $D_p$  are electron and hole diffusion constants. The density of electron  $\Delta I_n$  and hole  $\Delta I_p$  components of photocurrent

$$I_{ph} = \Delta I_n + \Delta I_p \quad (14)$$

must satisfy continuity equations:

$$\frac{\partial \Delta I_n}{\partial x} = q \times (R_n - g), \quad (15)$$

$$\frac{\partial \Delta I_p}{\partial x} = q \times (g - R_p), \quad (16)$$



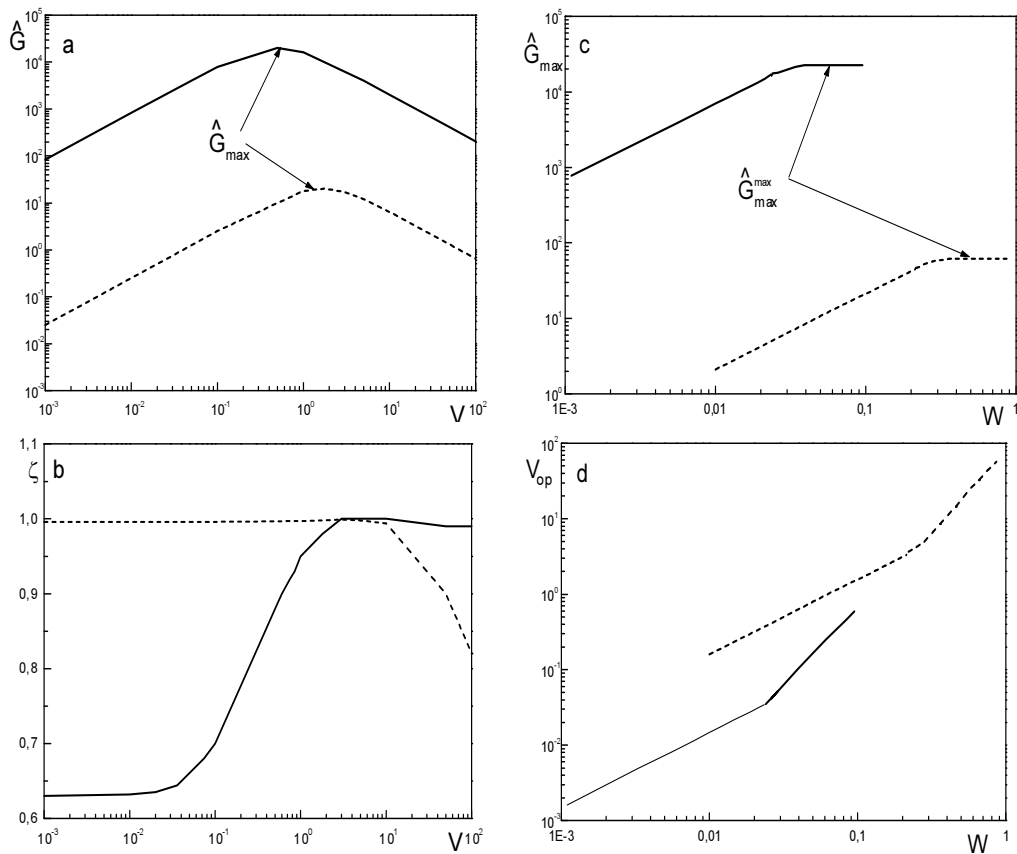
and also

$$\frac{\partial I_{ph}}{\partial x} = 0. \quad (17)$$

Let limit voltage be applied to sample by value

$$V = E_0 \times W \quad (18)$$

that allows to neglect by the dependence of  $\mu_n$  and  $\mu_p$  on electric field, where  $W$  is distance between current contact electrodes (see insert in Figure 1a).



**Figure 2.** Dependences: (a), photoelectric gain  $G = \hat{G}$  in point of maximal extremum of function  $G(N_\lambda)$  (see Figure 1a) on bias voltage across the sample  $V$  (distance between current contacts  $W = 10^{-1}$  cm); (b), ratio  $\zeta \equiv G_{appr} / G_{exact}$  on  $V$  at  $W = 10^{-1}$  cm, where  $\hat{G}_{appr}$  and  $\hat{G}_{exact}$  are approximate and exact values  $\hat{G}$ , respectively; (c), value  $\hat{G}_{max}$  on  $W$ , where  $\hat{G}_{max}$  is maximal value  $G$  for given  $W$  (see Figure 2a); (d), value  $V_{op}$  on  $W$ , where  $V_{op}$  is optimal voltage, at which  $G = G_{max}$  (see Figure 2c). Voltage  $V$  in V; length  $W$  in cm. Physical parameters of semiconductors and temperature are the same as in Figure 1. Solid curves *GaAs*, dashed curves *Si*

#### 4. Lifetime of excess charge carriers

Using distribution function of electrons over acceptor level states [12, 27], we can write neutrality equation

$$n_e + N_-^e = p_e + N_D \quad (19)$$

for nondegenerated semiconductor at thermodynamic equilibrium as follows:

$$N = n_t \times \frac{1 + \delta}{2 \times \delta^2} \times f(\delta), \quad (20)$$

where

$$\delta = \frac{N_-^e}{N_0^e}, f(\delta) = B + A \times \delta - \delta^2, \quad (21)$$

$$A = 2 \times \frac{N_D}{n_t}, B = 4 \times \frac{p_t}{n_t} = \left( \frac{2 \times n_i}{n_t} \right)^2, \quad (22)$$

$n_t$  and  $p_t$  are equilibrium concentrations of electrons and holes when Fermi level energy coincides with recombination level energy  $E_t$ , and  $n_i$  is intrinsic charge carriers' concentration. When derived (20), we have adopted that spin degeneracy factor of acceptor state is equal to 1/2 [2, 12, 14, 27]. In considered conditions

$$n_e = \frac{\delta}{2} \times n_t, p_e = \frac{2}{\delta} \times p_t. \quad (23)$$

From expressions (8), (9), (20), and (23), it follows that

$$\tau_n = \frac{2\delta^2}{f(\delta)} \times \frac{\theta \times f(\delta) + (1 + \delta) \times (\theta \times B + \delta)}{[\delta \times A + (2 + \delta) \times B + \delta^3] w_p \times n_t}, \quad (24)$$

$$\tau_p = \frac{2\delta}{f(\delta)} \times \frac{B + (A + \theta \times B) \times \delta + (\theta \times B + \delta) \times \delta^2}{[\delta \times A + (2 + \delta) \times B + \delta^3] w_p \times n_t}. \quad (25)$$

Expressions (20), (24), and (25) determine dependences  $\tau_n(N)$  and  $\tau_p(N)$  in parametric form. Figure 1b shows that, as usual, dependences  $\tau_n(N)$  and  $\tau_p(N)$  fall with increased  $N$ , but in some interval of concentration  $N$ , dependences can rise up sharply. Further, we give analytical solution of extremum problem for dependences  $\tau_n(N)$  and  $\tau_p(N)$  at  $\theta \geq 1$ , because hole is captured on attractive center and electron – on neutral.

#### 4.1. Mathematical analysis of hole lifetime

The analysis of equation

$$\frac{\partial}{\partial \delta} \times \frac{1}{\tau_p} = 0, \quad (26)$$

which determines extremum points of dependence  $\tau_p(N)$ , shows that well-defined non-monotonic behavior of this function occurs at

$$\xi_1 \equiv \frac{3}{\theta \times B} \ll 1 \ll \frac{1}{\xi_2} \equiv \frac{A^2}{4 \times B}, \xi_3 \equiv \frac{1}{\sqrt{A}} \ll 1. \quad (27)$$

To determine minimum point, let us set out equation (26) into the form

$$\delta^2 + 2 \times \theta \times B \times \delta - \theta \times A \times B \times [1 + \Lambda_{1p}(\delta)] = 0, \quad (28)$$

where absolute value of function

$$\begin{aligned} \Lambda_{1p}(\delta) = & \frac{B + (A + \theta \times B) \times \delta}{\theta \times A \times B \times \delta} - \frac{B}{A \times \delta} + \frac{B \times \delta^3 + (\delta^2 + B)[2 \times B + (A + B) \times \delta]}{\theta \times A \times B \times \delta^6} \times \\ & \times [B + (A + \theta \times B) \times \delta + \theta \times B \times \delta^2 + \delta^3] + \frac{B + A \times \delta - \delta^2}{\theta \times A \times \delta^5} \times \\ & \times \{A + (2 \times \theta - 1) \times B + 4 \times \theta \times B \times \delta + [3 + (A + B) \times \theta] \times \delta^2 - 2(\theta - 1) \times \delta^3\} \end{aligned} \quad (29)$$

is much less than unity at

$$\delta = \delta_{1p}^{(0)} \equiv -\theta \times B + \sqrt{\theta \times B \times (A + \theta \times B)}. \quad (30)$$

It means that the first root of equation (26)  $\delta_{1p}$  can be found by the method of successive iterations using  $\Lambda_{1p}(\delta_{1p})$  as small parameter. Zeroth-order approximation (30) leads to the formula for concentration of recombination centers  $N = N_{1p}$ , where dependence  $\tau_p(N)$  reaches its minimum  $\tilde{\tau}_p$  (Figure 1b):

$$N_{1p} = N_D + 2 \times \theta \times p_t - \sqrt{2 \times \theta \times p_t \times (N_D + 2 \times \theta \times p_t)}. \quad (31)$$

It follows from this formula that ratio  $N_{1p}/N_D$  increases from  $\frac{1}{2}$  when  $N_D < 2 \times \theta \times p_t$  to  $(1 - \sqrt{2 \times \theta \times p_t}) \cong 1$  when  $N_D > 2 \times \theta \times p_t$  with increased  $N_D$  (Figure 3a). Further, it will be demonstrated that extremum, like the maximum of dependence  $\tau_p(N)$  as well as  $\tau_n(N)$ , can occur only at values  $N$  closer to  $N_D$ . Therefore, in expression for upper limit of value  $N_{1p}$ , small correction has been remained, which is primal.

To determine maximum point of dependence  $\tau_p(N)$ , let us set out equation (26) into the form

$$[1 - \Lambda_{2p}(\delta)] \times \delta^2 = A + B, \quad (32)$$

where absolute value of function

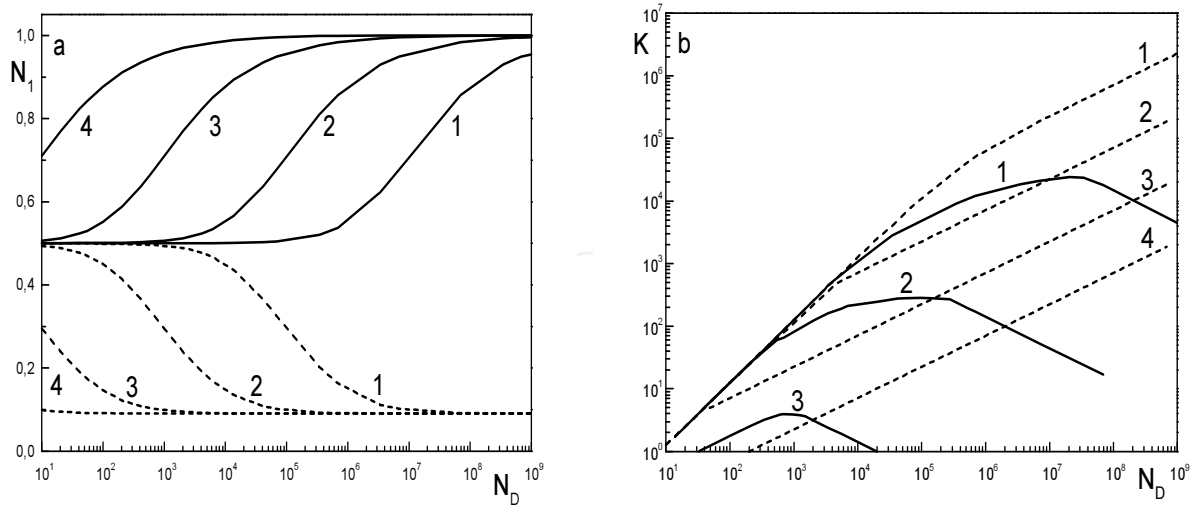
$$\begin{aligned} \Lambda_{2p}(\delta) = & \frac{1}{\theta \times \delta^4} \times [A + (2 \times \theta - 1) \times B + 3\delta^2 - 2 \times (\theta - 1) \times \delta^3] + \\ & + \frac{B + \delta^2}{\theta \times B \times \delta^5} \times [2 \times B + (A + B) \times \delta] \times \left[ 1 + \frac{\delta \times (\delta + 1) \times (\theta \times B + \delta)}{B + A \times \delta - \delta^2} \right] \end{aligned} \quad (33)$$

is much less than unity at

$$\delta = \delta_{2p}^{(0)} = \sqrt{A + B}. \quad (34)$$

It means that the second root of equation (26)  $\delta_{2p}$  can be found by the method of successive iterations using  $\Lambda_{2p}(\delta_{2p})$  as small parameter. It follows from relations (20) and (34) that concentration of recombination centers  $N = N_{2p}$ , where dependence  $\tau_p(N)$  reaches its maximum  $\hat{\tau}_p$  (Figure 1b), is determined in first-order approximation for small parameter  $\Lambda_{2p}(\delta_{2p})$  by expression

$$\frac{N_{2p}}{N_D} = 1 - \frac{1}{A + B} - \frac{\sqrt{A + B}}{A} \times \Lambda_{2p} \times (\sqrt{A + B}) \cong 1. \quad (35)$$



**Figure 3.** Dependences: point of minimum  $N_1$  of functions  $\tau_p(N)$  и  $\tau_n(N)$  (a) and ratio  $K$  between maximal to minimal charge carriers' lifetimes (b) on concentration of shallow donors  $N_D$  at different locations of recombination level. Solid curves, holes; dashed curves, electrons. Ratio  $n_t/n_i$  values: curve 1,  $10^{-4}$ ; curve 2,  $10^{-2}$ ; curve 3, 1; curve 4,  $10^2$ . Concentration  $N_1$  is measured in units  $N_D$ , concentration  $N_D$  in units  $n_i$ . Adopted:  $\theta = 10^2$

In particular

$$\frac{N_{2p}}{N_D} = \begin{cases} 1 - \frac{n_t}{2 \times N_D} \times \left( 1 + \frac{2}{\theta} + \frac{\sqrt{2 \times N_D \times n_t}}{\theta \times p_t} \right), & \text{for } N_D \gg 2 \times p_t \\ 1 - \left( \frac{2 \times n_i}{N_D} \right)^2, & \text{for } 2n_i \ll N_D \ll 2p_t \end{cases} \quad (36)$$

It follows from (22), (25), (27), (30), and (34) that

$$K_p(N_D) \equiv \frac{\hat{\tau}_p}{\tilde{\tau}_p} = \begin{cases} \frac{N_D}{8 \times n_i}, & \text{for } 8 \times n_i \ll N_D \ll 2 \times p_t \\ \sqrt{\frac{N_D}{32 \times n_i}}, & \text{for } N_D \ll 2\theta \times p_t \ll \theta \times N_D \\ \frac{\sqrt{2 \times \theta \times p_t}}{\sqrt{N_D \times n_t}}, & \text{for } \sqrt{2 \times N_D \times n_t} \ll 2 \times \theta \times p_t \ll N_D \\ \approx 1, & \text{for } 2 \times \theta \times p_t \ll \sqrt{2 \times N_D \times n_t} \end{cases} \quad (37)$$

Relation (37) shows that function  $K_p(N_D)$  is non-monotonic and can vary by several orders of magnitude (Figure 3b). Value

$$\hat{\tau}_p = \left( 1 + \frac{\sqrt{2} \times \theta \times p_t}{\sqrt{N_D \times n_t + 2 \times n_i^2}} \right) \times (N_D \times w_p)^{-1} \quad (38)$$

increases with decreasing recombination level energy  $E_t$  (Figure 4a).

The dependence of  $\hat{\tau}_p$  on temperature  $T$  is determined by the location of recombination level in forbidden gap of semiconductor (Figure 4b). Value  $\hat{\tau}_p$  decreases with lowering temperature if  $E_t \geq E_g/2$  and increases if  $E_t \leq E_g/3$  (value  $E_t$  is measured from top of valence band and  $E_g$  is the energy gap of semiconductor). If  $E_g/3 < E_t < E_g/2$ , then dependence  $\hat{\tau}_p$  has maximum value at

$$T = \tilde{T} \equiv \frac{E_t}{k} \times \ln^{-1} \left( 2 \times \frac{N_v}{N_D} \times \frac{E_g - 2 \times E_t}{3 \times E_t - E_g} \right), \quad (39)$$

where  $k$  is Boltzmann constant and  $N_v$  is effective density of states in valance band.

#### 4.2. Mathematical analysis of electron lifetime

The analysis of equation

$$\frac{\partial}{\partial \delta} \times \frac{1}{\tau_n} = 0, \quad (40)$$

which determines extremum points of dependence  $\tau_n(N)$ , shows that well-defined non-monotonic behavior of this function occurs at

$$2 \times \xi_3 \ll 1, \xi_4 = 2 \times \frac{\sqrt{B}}{A} \ll 1. \quad (41)$$

To determine minimum point let us set out equation (40) into the form

$$(\theta - 1) \times \delta^2 - 2 \times \theta \times (A + B) \times \delta + \theta \times A \times (A + B) \times [1 + \Lambda_{1n}(\delta)] = 0, \quad (42)$$

where absolute value of function

$$\begin{aligned} \Lambda_{1n}(\delta) = & \frac{(\theta - 1) \times B}{\theta \times A \times (A + B)} - \frac{2 \times B + A \times \delta}{\theta \times A \times (A + B) \times \delta} \times \\ & \times \left[ \left( 1 + \frac{2 \times \theta \times B}{\delta} \right) \times \left( 1 + \frac{A + B}{\delta^2} + \frac{2 \times B}{\delta^3} \right) + \frac{1}{\delta} \times \left( A + B + \frac{2 \times B}{\delta} \right) \times \left( \theta \times \frac{A + B}{\delta} - \theta + 1 \right) \right] + \\ & + \frac{B + A \times \delta - \delta^2}{\theta \times A \times (A + B) \times \delta} \left[ 2 + (\theta - 1) \times \frac{A + B}{\delta} + \frac{6 \times \theta \times B}{\delta} + \frac{2 \times B}{\delta^2} \times \left( 2 \times \theta - 2 - \frac{1}{\delta} \right) \right] \end{aligned} \quad (43)$$

is much less than unity at

$$\delta = \delta_{1n}^{(0)} \equiv \frac{\sqrt{\theta \times (A+B)}}{\theta-1} \times \left( \sqrt{\theta \times (A+B)} - \sqrt{A+\theta \times B} \right). \quad (44)$$

It means that the first root of equation (40)  $\delta_{1n}$  can be found by the method of successive iterations using  $\Lambda_{1n}(\delta_{1n})$  as small parameter. Zeroth-order approximation (44) leads to the formula for concentration of recombination centers  $N = N_{1n}$  where dependence  $\tau_n(N)$  reaches its minimum  $\hat{\tau}_n$ :

$$N_{1n} = \frac{\sqrt{N_D + 2 \times \theta \times p_t}}{\theta-1} \times \left( \sqrt{\theta \times (N_D + 2 \times p_t)} - \sqrt{N_D + 2 \times \theta \times p_t} \right). \quad (45)$$

It follows, from this formula, that ratio  $N_{1n}/N_D$  decreases from  $1/2$  when  $N_D \ll 2 \times \theta \times p_t$  to  $1/(\sqrt{\theta} + 1)$  when  $N_D \gg 2 \times \theta \times p_t$  with increased  $N_D$  (Figure 3a).

We can transform equation (40) to form (32) where  $\Lambda_{2p}(\delta)$  will be replaced by function:

$$\begin{aligned} \Lambda_{2n}(\delta) = & \frac{2 \times \delta}{A} + \frac{2 \times B}{A} \times \frac{\theta \times (A+B) \times \delta + 2 \times \theta \times B + \delta}{\theta \times (A+B) \times \delta^4} \times \left( 2 \times A + B + \frac{2 \times B}{\delta} \right) + (2 \times \theta \times B + \delta) \times \\ & \times \frac{A \times (A+B+\delta^2) + 2 \times B \times \delta}{\theta \times A \times (A+B) \times \delta^3} - 2 \times \frac{B + A \times \delta - \delta^2}{\theta \times A \times (A+B) \times \delta^4} \times (\delta^3 + 3 \times \theta \times B \times \delta^2 - 2 \times B) - (\theta-1) \times \\ & \times \left[ \frac{2 \times B \times [4 \times B + (4 \times A + B) \times \delta + (\delta-2) \times \delta^2] + 2 \times A \times (A+B) \times \delta^2 + \delta \times (B - \delta^2) \times (A+B - \delta^2)}{\theta \times A \times (A+B) \times \delta^4} \right]. \end{aligned} \quad (46)$$

At value  $\delta = \delta_{2n}$ , where dependence  $\tau_n(N)$  has its maximum  $\hat{\tau}_n$ , absolute value  $|\Lambda_{2n}(\delta)| < 1$ . Therefore, in zeroth-order approximation for small parameter  $\Lambda_{2n}(\delta_{2n})$ , value  $\delta_{2n} = \delta_{2p}^{(0)}$  and concentration  $N = N_{1n}$ , where  $\tau_n(N) = \hat{\tau}_n$  equals to  $N_D$  (as for holes). And

$$K_n(N_D) \equiv \frac{\hat{\tau}_n}{\bar{\tau}_n} = \begin{cases} \frac{N_D}{8 \times n_i}, & \text{for } 2 \times n_i \ll N_D \ll 2 \times p_t \\ \frac{1}{\kappa} \times \sqrt{\frac{N_D}{2 \times n_t}}, & \text{for } N_D \gg 2 \times p_t \end{cases} \quad (47)$$

where  $\kappa=4$  at  $\theta=1$  and  $\kappa=1$  at  $\theta \gg 1$ . It follows from relation (47) that function  $K_n(N_D)$ , in contrast to  $K_p(N_D)$ , grows monotonically with increased  $N_D$  and this growth can be many orders of magnitude (Figure 4b).



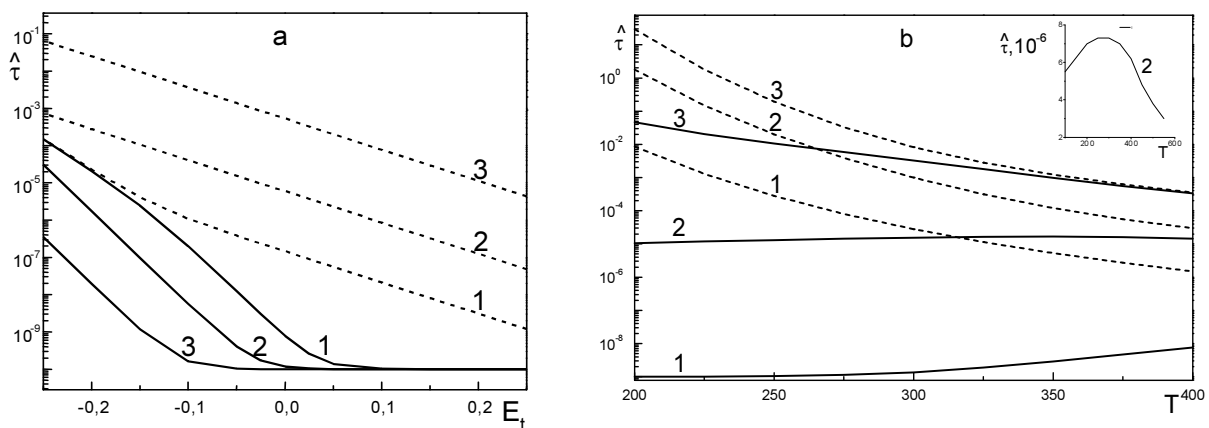
Value

$$\hat{\tau}_n = \sqrt{\frac{N_D + 2 \times p_t}{2 \times n_i}} \times (N_D \times w_n)^{-1} \quad (48)$$

increases, as for  $\hat{\tau}_p$ , with the decrease of recombination level energy (Figure 4a) and, in contrast to  $\hat{\tau}_p$ , always falls with temperature rise (Figure 4b).

### 4.3. Physical interpretation

Let's explain physical mechanisms of the above regularities.



**Figure 4.** Dependences of maximal lifetime of holes  $\hat{\tau}_p(N)$  (solid lines) and electrons  $\hat{\tau}_n(N)$  (dashed lines) on recombination level energy  $E_t$  (eV) for Ge (1), Si(2), and GaAs (3) at  $T = 300$  K (a) and on temperature  $T$  (K) for Si at different values  $E_t$  (b): curve 1, 0; curve 2, (-101/152); curve 3, (-3/4). Recombination level energy is reckoned from the middle of forbidden gap. Physical parameters for Ge, Si, and GaAs are obtained from monograph [3]. Adopted:  $N_D = 10^{15} \text{ cm}^{-3}$ ,  $w_n = 10^{-8} \text{ cm}^3/\text{s}$ ,  $\theta = 10^2$

#### 4.3.1. Hole lifetime

Reciprocal hole lifetime

$$\tau_p^{-1} = \tau_{p1}^{-1} + \tau_{p2}^{-1} + \tau_{p3}^{-1} \quad (49)$$

consists of three partial components.

First component

$$\tau_{p1}^{-1} = w_p \times N_-^e = \frac{w_p \times n_t}{2} \times \frac{B + A \times \delta - \delta^2}{\delta} \quad (50)$$

corresponds to the change of capture rate of holes  $\Delta p \times w_p \times N_-^e = \Delta p / \tau_{p1}$  caused only by deviation of hole concentration from its equilibrium value (capture of excess holes  $\Delta p$  at equilibrium trapping centers  $N_-^e$ ).

Second component

$$\tau_{p2}^{-1} = -w_p \times p_t \times \frac{2}{\delta} \times \frac{(\delta \times \theta - 1) \times (B + A \times \delta - \delta^2)}{A \times \delta + B \times [1 + \delta \times \theta \times (1 + \delta)] + \delta^3} \quad (51)$$

corresponds to the change of capture rate of holes  $p_e \times w_p \times \Delta N_- = \Delta p / \tau_{p2}$  caused only by deviation of concentration of hole trapping centers from its equilibrium value (capture of equilibrium holes  $p_e$  at nonequilibrium trapping centers  $\Delta N_-$ ).

Third component

$$\tau_{p3}^{-1} = \delta \times \tau_{p2}^{-1} \quad (52)$$

corresponds to the change of thermal emission rate of holes from impurity level states into valence band  $2 \times p_t \times w_p \times \Delta N_0 = \Delta p / \tau_{p3}$  caused by deviation of concentration of hole generation centers from its equilibrium value (thermal emission of holes from nonequilibrium centers  $\Delta N_0$ ).

Lifetime  $\tau_{p1}$  can be interpreted as capture time of excess holes by equilibrium traps, lifetime  $\tau_{p2}$  can be interpreted as relaxation time of excess holes due to capture of equilibrium holes by nonequilibrium traps, and lifetime  $\tau_{p3}$  can be interpreted as time of thermal emission of holes from nonequilibrium centers.

If conditions (27) are fulfilled and  $N < N_D$ , then recombination centers are almost completely filled with electrons ( $\delta \equiv N_-^e / N_0^e > 1$ ). For this reason, even if  $\theta \equiv w_p / w_n = 1$ , capture time of hole  $\tau_{p1}$  is much shorter than capture time of electron  $\tau_{n1}$  for the relevant equilibrium trapping centers. In other words, equilibrium traps capture holes much more intensively than electrons. Therefore, the generation of excess free charge carriers initiates the formation of additional nonequilibrium centers of thermal generation of holes and, simultaneously, reducing concentration of trapping centers of electrons ( $\Delta N_0 = -\Delta N_- > 0$ ). This change of charge state of recombination impurity atoms results in negative values of components  $1/\tau_{p2}$  and  $1/\tau_{p3}$  in expression (36); moreover  $|\tau_{p3}| < |\tau_{p2}|$ , because  $\delta > 1$  at  $N < N_D$ . This means that hole lifetime  $\tau_p$  exceeds capture time of holes  $\tau_{p1}$  at equilibrium traps due to dominating thermal emission of holes from relevant nonequilibrium centers. As long as  $N < N_D$ , concentration  $N_-^e \cong N$  of equilibrium capture centers of holes grows with increased  $N$ , but concentration  $N_0^e < N$  of capture centers of electrons still remains low. Therefore, concentration  $\Delta N_0$  of nonequilibrium centers of hole thermal emission increases as well. This increase causes faster

decreasing  $|\tau_{p3}|$  than decreasing  $\tau_{p1}$ . As a result, starting with concentration  $N = N_{1p}$ , the rate of hole thermal emission from nonequilibrium centers and the capture rate of nonequilibrium holes at equilibrium traps become closer to each other. For this reason,  $\tau_p$  starts to grow (Figure 1b).

When  $N$  becomes larger than  $N_D$ , the concentration  $N_-^e$  of equilibrium hole capture centers practically stabilizes, while concentration  $N_0^e$  of equilibrium electron capture centers grows with the increase of recombination centers' concentration ( $N_-^e \cong N_D, N_0^e \cong N - N_D$  at  $n_t/2 \ll N - N_D \ll N_D^2/2p_t$ ). This means that the ratio  $\tau_{p1}/\tau_{n1}$  increases with increasing  $N$ . For this reason, concentration of nonequilibrium hole thermal emission centers decreases, and concentration of hole traps  $N_-$  increases. As a result,  $\tau_p(N)$  decreases with increased  $N$  (Figure 1b). When  $N$  prevails  $N_D^2/2p_t$ , the concentration  $N_-^e$  of equilibrium hole capture centers grows again with increased  $N$  due to thermal emission of electrons from valence band to impurity level ( $N_-^e \cong \sqrt{2N \times p_t}$ ). However, concentration of equilibrium capture centers of electrons grows much faster ( $N_0^e \cong N$ ). Therefore, the decrease of  $\tau_p(N)$  continues. As it is seen from (36)-(39),  $\tau_p$  becomes less than  $\tau_{p1}$ , when product  $\delta \times \theta$  becomes less than unity.

As shown above, minimum point  $N = N_{1p}$  of dependence  $\tau_p(N)$  shifts toward  $N_D$  with growth  $N_D$  (Figure 3a). The main reason is that equilibrium electrons are being captured at centers of hole thermal emission and decreased concentration  $\Delta N_0$  of these centers. The higher the concentration of equilibrium electrons  $n_e$ , the more  $\Delta N_0$  decreases. Concentration  $n_e$  grows with increased  $N_D$ . When  $N$  ascends, then  $n_e$  descends and  $N_-^e$  increases that causes increased  $\Delta N_0$  at  $N < N_D$ . In other words, decreased  $\Delta N_0$  with increased  $N_D$  is compensated by increased  $\Delta N_0$  with increased  $N$ . This is the reason why the greater the  $N_D$ , the closer the  $N_{1p}$  to  $N_D$ .

For the same reasons, non-monotonic dependence  $\tau_p$  on  $N$  cancels out, as shown above (Figure 3b), at  $N_D > 2 \times (\theta \times p_t)^2 / n_t$  (increased  $\Delta N_0$  with increasing  $N$  is not able to compensate decreasing  $\Delta N_0$  with increasing  $N_D$ ).

Non-monotonic character of dependence  $\tau_p$  on  $N$  does not occur and at low concentrations  $N_D$  {see inequities (27), expressions (22), and Figure 3b}, when equilibrium electron population at recombination level is determined mostly by electron-hole transitions between that level and free bands. In this case, values  $\delta$  cannot provide prevailing growth of hole thermal emission rate from nonequilibrium centers over the growth of capture rate of nonequilibrium holes at equilibrium hole traps with increasing  $N$ .

Maximal value of ratio  $K_p \equiv \hat{\tau}_p / \check{\tau}_p$  is achieved at  $N_D \cong 2 \times \theta \times p_t$  and equals to approximately

$$(K_p)_{\max} \cong \sqrt{\frac{\theta \times p_t}{n_t}}. \quad (53)$$

Note that with increasing energy  $E_t$  of recombination level, non-monotonicity of dependence  $\tau_p(N)$  fades out (Figure 3b) and then cancels out absolutely. This is caused by the increase in concentration  $n_e$  of equilibrium electrons and decrease in value  $\delta$  and fall of the probability of hole thermal emission from recombination level into valence band with increasing energy of recombination level referred to the top of valence band. For the same reason, value  $\hat{\tau}_p$  decreases with increasing  $E_t$  (Figure 4a).

The character of dependence  $\hat{\tau}_p$  on temperature (Figure 4b) is determined by the following dependences on temperature:

$$\delta_{2p}(T) = \frac{1}{n_i(T)} \times \sqrt{2 \times N_D \times p_t(T) + 4 \times p_t^2(T)}, \quad (54)$$

$$p_t(T) = N_v \times \exp\left(-\frac{E_t}{kT}\right), n_e(T) = \frac{\delta_{2p}(T) \times n_i^2(T)}{2 \times p_t(T)}. \quad (55)$$

Values  $p_t(T)$  and  $n_e(T)$  increase always with temperature  $T$  rise. Increased  $p_t$  means increasing probability of thermal emission of hole from recombination center into valence band. Therefore, the above-mentioned process facilitates increasing  $\hat{\tau}_p$  with  $T$  rise. At the same time, growth  $n_e(T)$  facilitates decreasing  $\hat{\tau}_p$  with  $T$  rise due to decreasing concentration of nonequilibrium centers  $\Delta N_0$  of hole thermal emission.

Value  $\delta_{2p}$  decreases with  $T$  rise at  $E_t \leq E_g/3$  due to approaching  $N_{2p}$  closer and closer to  $N_D$  (see expression (36)). Value  $\delta_{2p}$  decreases also at  $E_t \geq E_g/2$  up to temperature at which non-monotonic dependences  $\tau_p$  and  $\tau_n$  on  $N$  cancel out. Falling  $\delta_{2p}$  decreases  $\Delta N_0$  that facilitates decreasing  $\hat{\tau}_p$  with  $T$  rise. When  $E_t \geq E_g/2$ , then  $p_t(T)$  increases faster and  $\delta_{2p}(T)$  falls and  $n_e(T)$  grows. As a result,  $\hat{\tau}_p$  increases with temperature rise (Figure 4b). If  $E_t \leq E_g/3$ , then increased  $p_t$  with temperature rise cannot compensate decreased  $\delta_{2p}(T)$  and growth  $n_e(T)$ . As a result,  $\hat{\tau}_p$  decreases with temperature rise (Figure 4b). If  $E_g/3 < E_t < E_g/2$ , then at  $T < \tilde{T}$ , dependence  $\hat{\tau}_p(T)$  will be increasing, and at  $T > \tilde{T}$  dependence  $\hat{\tau}_p(T)$  will be falling for the same reasons that in previous cases (see expression (39) and insert in Figure 4b).

#### 4.3.2. Electron lifetime

By analogy with hole lifetime, reciprocal electron lifetime consists of three partial components:

$$\tau_n^{-1} = \tau_{n1}^{-1} + \tau_{n2}^{-1} + \tau_{n3}^{-1} \quad (56)$$

First component

$$\tau_{n1}^{-1} = w_n \times N_0^e = \frac{w_n \times n_t}{2} \times \frac{B + A \times \delta - \delta^2}{\delta^2} \quad (57)$$

corresponds to the change of electron capture rate  $\Delta n \times w_p \times N_0^e = \Delta n / \tau_{n1}$  caused by deviation of electron concentration from equilibrium value (capture of excess electrons  $\Delta n$  on equilibrium traps  $N_0^e$ ).

Second component

$$\tau_{n2}^{-1} = \frac{w_n \times n_t}{2} \times \frac{(\delta \times \theta - 1) \times (B + A \times \delta - \delta^2)}{\theta \times (B + A \times \delta - \delta^2) + (1 + \delta) \times (\theta \times B + \delta)} \quad (58)$$

corresponds to the change of electron capture rate  $n_e \times w_n \times \Delta N_0 = \Delta n / \tau_{n2}$  caused solely by deviation of concentration of electron capture centers from equilibrium value (capture of equilibrium electrons  $n_e$  on nonequilibrium capture centers  $\Delta N_0$ ).

Third component

$$\tau_{n3}^{-1} = (\delta \times \tau_{n2})^{-1} \quad (59)$$

corresponds to the change of thermal emission rate of electrons from impurity level into valence band  $(1/2) \times n_t \times w_n \times \Delta N_- = -\Delta n / \tau_{n3}$  caused by deviation of concentration of electron thermal emission centers from equilibrium value (thermal emission of electrons from nonequilibrium centers  $\Delta N_-$ ).

Times  $\tau_{n1}$ ,  $\tau_{n2}$  and  $(-\tau_{n3})$  have physical sense similar to times  $\tau_{p1}$ ,  $\tau_{p2}$  and  $(-\tau_{p3})$ , respectively.

Value  $\delta \gg 1$  as long as  $N < N_D$ , and hence ratio  $\tau_{p1} / \tau_{n1} \ll 1$ . Therefore, the occurrence of excess free charge carriers leads to the formation of additional (nonequilibrium) capture centers of electrons and, at the same time, decrease in concentration ( $\Delta N_0 = -\Delta N_- > 0$ ) of electrons' generation centers. Partial components  $1 / \tau_{n2}$  and  $1 / \tau_{n3}$  are positive values at such change of charge state of recombination centers; moreover  $\tau_{n2} < \tau_{n3}$  because of  $\delta > 1$  at  $N < N_D$ . It means that, due to preferable capture of equilibrium electrons on nonequilibrium traps, lifetime of electrons  $\tau_n$  is shorter, than capture time of electrons on equilibrium traps. With further increased  $N$ , the number of equilibrium capture centers of holes grows, but the number of equilibrium capture centers of electrons remains still small. As a result, concentration  $\Delta N_0$  of nonequilibrium capture centers of electrons increases. For this reason, starting from concentration  $N = N_{1n}$ , capture rate of equilibrium electrons on nonequilibrium traps becomes higher than capture rate of nonequilibrium electrons on equilibrium traps. In other words, partial component  $1 / \tau_{n2}$  becomes critical component defining reciprocal lifetime of electrons  $1 / \tau_n$ .

Component  $1/\tau_{n2}$  falls with growth  $N$  due to the decrease in concentration  $n_e$  of equilibrium electrons, so  $\tau_n \approx \tau_{n2}$  and increases with growth  $N$  (Figure 1b).

At values  $N$  greater than  $N_D$ , the ratio  $\tau_{p1}/\tau_{n1}$  increases with increasing  $N$ . This, again, leads to decreasing concentration of nonequilibrium capture centers of electrons with increasing  $N$ . Value  $n_e$  continues to fall as well. As a result, partial component  $1/\tau_{n1}$  becomes critical component defining reciprocal electron lifetime  $1/\tau_n$ , and therefore,  $\tau_n(N)$  falls with increasing  $N$  (Figure 1b).

As shown above, in contrast to dependence  $\tau_p(N)$ , ratio  $N_{1n}/N_D$  decreases (Figure 3a) and ratio  $K_n \equiv \hat{\tau}_n/\check{\tau}_n$  always increases monotonically with increasing  $N_D$  (Figure 3b). Such regularities are caused by increased  $n_e$  with increasing  $N_D$ . Because of this, capture rate of equilibrium electrons at nonequilibrium traps becomes greater than capture rate of nonequilibrium electrons at equilibrium traps at lower concentrations  $\Delta N_0$ , i.e., at lower values of ratio  $N/N_D$ . In contrast to the situation with holes, here, decreasing  $\Delta N_0$  with increased  $N_D$  is compensated by increasing  $n_e$ .

Similar to the behavior of hole lifetime, non-monotonic dependence  $\tau_n(N)$  fades gradually and then cancels out (Figure 3b) with decreasing  $N_D$  or increasing  $E_t$ . First regularity is caused by decreased  $n_e$  and  $\delta = 2n_e/n_i$  with decreasing  $N_D$ . Second regularity is caused by decreasing  $\delta$  and, hence,  $\Delta N_0$ , with increasing energy  $E_t$  of recombination level. In this case, however, due to growth  $n_e$ , non-monotonicity of dependence  $\tau_n(N)$  cancels out at larger values  $E_t$  than in the case of holes.

Due to decreasing  $\delta_{2n}$  with increasing  $E_t$ , value  $\hat{\tau}_n$  decreases as well (Figure 4a). The type of dependence  $\hat{\tau}_n$  on temperature (Figure 4b) is determined only by dependence  $\delta_{2n}(T)$ , because in maximum point  $\tau_n = \tau_{n1}/2 \sim 1/N_0^e \sim \delta_{2n}$ . In zeroth approximation,  $\delta_{2n}(T)$  coincides with  $\delta_{2p}(T)$ , determined by expression (56). Therefore,  $\hat{\tau}_n$  decreases always with temperature rise (Figure 4b).

## 5. Relation between concentrations of photoholes and photoelectrons outside approximation of quasi-neutrality

Note first, from formulas (10), (11), (20), and (23) follow

$$a_n = \frac{\varepsilon}{8\pi \times q} \times \frac{(1+\delta) \times f(\delta) \times w_p \times n_i}{\theta \times f(\delta) + (1+\delta) \times (\theta \times B + \delta)}, \quad (60)$$

$$a_p = \frac{\varepsilon}{2\pi \times q} \times \frac{(1 + \delta^{-1}) \times f(\delta) \times w_p \times p_i}{B + (A + \theta \times B) \times \delta + (\theta \times B + \delta) \times \delta^2}. \quad (61)$$

Differentiating (12) with respect to  $x$ , we obtain

$$\frac{\partial \Delta E}{\partial x} = \frac{1}{q \times \mu_n \times n_e} \times \frac{\partial \Delta I_n}{\partial x} - \frac{E_0}{n_e} \times \frac{\partial \Delta n}{\partial x} - \frac{D_n}{\mu_n n_e} \times \frac{\partial^2 \Delta n}{\partial x^2}. \quad (62)$$

Recall that  $\Delta E \equiv E(x) - E_0$  is the change of electric field intensity caused by deviation of concentrations of free charge carriers and their capture centers from equilibrium values by reason of band-to-band absorption of optical radiation:  $E(x)$  and  $E_0$  are electric field intensities with and without illumination.

From equation (16) and formula (5), we find

$$\frac{1}{q} \times \frac{\partial \Delta I_n}{\partial x} = \frac{\Delta n}{\tau_n} + a_n \times \frac{\partial \Delta E}{\partial x} - g. \quad (63)$$

Eliminating  $\frac{1}{q} \times \frac{\partial \Delta I_n}{\partial x}$  from equations (62) and (63), we obtain that

$$\frac{\partial \Delta E}{\partial x} = \frac{1}{1 - \tilde{\xi}_n} \times \left[ \frac{1}{\mu_n \times n_e} \times \left( \frac{\Delta n}{\tau_n} - g \right) - \frac{E_0}{n_e} \times \frac{\partial \Delta n}{\partial x} - \frac{D_n}{\mu_n \times n_e} \times \frac{\partial^2 \Delta n}{\partial x^2} \right], \quad (64)$$

where

$$\tilde{\xi}_n = \frac{a_n}{\mu_n \times n_e}. \quad (65)$$

Eliminating  $\frac{\partial \Delta E}{\partial x}$  from equations (7) and (64) and taking into account expressions (23) and (65), we find that relation between concentrations of excess holes  $\Delta p$  and electrons  $\Delta n$  is determined by the following formula:

$$\Delta p = \frac{\tau_p}{\tau_n} \times \Delta n + \chi \times \left( \frac{\tau_p}{\tau_n} \times \Delta n - g \times \tau_p - \mu_n \times \tau_p \times E_0 \times \frac{\partial \Delta n}{\partial x} - D_n \times \tau_p \times \frac{\partial^2 \Delta n}{\partial x^2} \right), \quad (66)$$



where

$$\chi = \frac{a_n + a_p}{\mu_n \times n_t \times \frac{\delta}{2} - a_n}. \quad (67)$$

In quasi-neutrality approximation  $a_n = a_p = 0$ ; therefore, in this case,

$$\Delta p = \frac{\tau_p}{\tau_n} \times \Delta n. \quad (68)$$

## 6. Derivation of equation for distribution function of photoexcited charge carriers' concentration outside quasi-neutrality

From expressions (12)–(14), it follows that

$$\Delta E = \frac{I_{ph} - q \times (\mu_n \times \Delta n + \mu_p \times \Delta p) \times E_0 + q \times D_p \times \frac{\partial \Delta p}{\partial x} - q \times D_n \times \frac{\partial \Delta n}{\partial x}}{q \times (\mu_n \times n_e + \mu_p \times p_e)}. \quad (69)$$

Plugging this expression for intensity  $\Delta E$  of photoinduced electric field in (12) and taking into account (17), we obtain

$$\Delta I_n = \frac{b \times n_e \times I_{ph}}{b \times n_e + p_e} - \frac{n_e \times \Delta p - p_e \times \Delta n}{b \times n_e + p_e} \times q \times \mu_n \times E_0 + \left( n_e \times \frac{\partial \Delta p}{\partial x} + p_e \times \frac{\partial \Delta n}{\partial x} \right) \times \frac{q \times D_n}{b \times n_e + p_e}, \quad (70)$$

where  $b = \mu_n / \mu_p = D_n / D_p$ . Formulas (17) and (70) allow writing

$$\frac{1}{q} \times \frac{\partial \Delta I_n}{\partial x} = - \frac{n_e \times \frac{\partial \Delta p}{\partial x} - p_e \times \frac{\partial \Delta n}{\partial x}}{b \times n_e + p_e} \times \mu_n \times E_0 + \left( n_e \times \frac{\partial^2 \Delta p}{\partial x^2} + p_e \times \frac{\partial^2 \Delta n}{\partial x^2} \right) \times \frac{D_n}{b \times n_e + p_e}. \quad (71)$$

Plugging relation (66) between  $\Delta p$  and  $\Delta n$  in (71), we find that

$$\begin{aligned} \frac{1}{q} \times \frac{\partial \Delta I_n}{\partial x} = & - \left( \mu_n^a + \frac{\xi}{1 - \xi_n} \times \frac{\tau_p}{\tau_n} \times \mu_p \right) \times E_0 \times \frac{\partial \Delta n}{\partial x} + \\ & + \left( D_n^a + \frac{\xi}{1 - \xi_n} \times \left( \tau_p \times \mu_p \times \mu_n \times E_0^2 + \frac{\tau_p}{\tau_n} \times D_p \right) \right) \times \frac{\partial^2 \Delta n}{\partial x^2} + \\ & + \frac{\xi}{1 - \xi_n} \left( -\tau_p \times \frac{\mu_p}{\mu_n} \times D_n^2 \times \frac{\partial^4 \Delta n}{\partial x^4} + \tau_p \times \mu_p \times E_0 \times \frac{\partial g}{\partial x} - \tau_p \times D_p \times \frac{\partial^2 g}{\partial x^2} \right), \end{aligned} \quad (72)$$

where

$$D_n^a = \frac{n_e \times \tau_p + p_e \times \tau_n}{(p_e + b \times n_e) \times \tau_n} \times D_n = \frac{\delta^2 \times \tau_p + B \times \tau_n}{(B + b \times \delta^2) \times \tau_n} \times D_n, \quad (73)$$

$$\mu_n^a = \frac{n_e \times \tau_p - p_e \times \tau_n}{(p_e + b \times n_e) \times \tau_n} \times \mu_n = \frac{\delta^2 \times \tau_p - B \times \tau_n}{(B + b \times \delta^2) \times \tau_n} \times \mu_n. \quad (74)$$

Formulas (73) and (74) are, none other than, well-known (in quasi-neutrality approximation) expressions for ambipolar diffusion constant  $D_n^a$ , and ambipolar mobility  $\mu_n^a$  of electrons and dimensionless parameter

$$\xi = \frac{a_n + a_p}{\mu_n \times n_e + \mu_p \times p_e} = \frac{2 \times (a_n + a_p) \times \delta}{(\delta^2 \times \mu_n + B \times \mu_p) \times n_t} \quad (75)$$

is much less than unity (see below). On the other hand, as it follows from equations (63) and (64),

$$\frac{1}{q} \times \frac{\partial \Delta I_n}{\partial x} = \frac{1}{1 - \xi_n} \times \left( \frac{\Delta n}{\tau_n} - g \right) - \frac{\xi_n}{1 - \xi_n} \times \left( \mu_n \times E_0 \times \frac{\partial \Delta n}{\partial x} + D_n \times \frac{\partial^2 \Delta n}{\partial x^2} \right). \quad (76)$$

Equating (72) and (76), we obtain equation

$$\begin{aligned} \xi \times D_n \times D_p \times \tau_p \times \frac{\partial^4 \Delta n}{\partial x^4} - & \left[ (1 - \xi_n) \times D_n^a + \xi \times \tau_p \times \mu_p \times \mu_n \times E_0^2 + \xi \times \frac{\tau_p}{\tau_n} \times D_p + \xi_n \times D_n \right] \times \frac{\partial^2 \Delta n}{\partial x^2} + \\ & + \left[ (1 - \xi_n) \times \mu_n^a + \xi \times \frac{\tau_p}{\tau_n} \times \mu_p - \xi_n \times \mu_n \right] \times E_0 \times \frac{\partial \Delta n}{\partial x} + \frac{\Delta n}{\tau_n} = g + \xi \times \tau_p \times \left( \mu_p \times E_0 \times \frac{\partial g}{\partial x} - D_p \times \frac{\partial^2 g}{\partial x^2} \right). \end{aligned} \quad (77)$$

Because

$$D_n - D_n^a = \frac{b - \tau_p / \tau_n}{b \times n_e + p_e} \times n_e \times D_n, \mu_n^a + \mu_n = \frac{b + \tau_p / \tau_n}{b \times n_e + p_e} \times n_e \times \mu_n, \quad (78)$$

then

$$(D_n - D_n^a) \times \tilde{\xi}_n + \xi \times \frac{\tau_p}{\tau_n} \times D_p = \xi_n \times D_n + \xi_p \times \frac{\tau_p}{\tau_n} \times D_p,$$

$$\xi \times \frac{\tau_p}{\tau_n} \times \mu_p - (\mu_n^a + \mu_n) \times \tilde{\xi}_n = \xi_p \times \frac{\tau_p}{\tau_n} \times \mu_p - \xi_n \times \mu_n, \quad (79)$$

where

$$\xi_n = \frac{a_n}{\mu_n n_e + \mu_p p_e} \ll 1, \xi_p = \xi - \xi_n = \frac{a_p}{\mu_n n_e + \mu_p p_e} \ll 1. \quad (80)$$

Therefore, we can rewrite equation (77) as follows:

$$Q \times \frac{\partial^4 \Delta n}{\partial x^4} - D \times \frac{\partial^2 \Delta n}{\partial x^2} + \mu \times E_0 \times \frac{\partial \Delta n}{\partial x} + \frac{\Delta n}{\tau_n} = g + \xi \times \tau_p \times \left( \mu_p \times E_0 \times \frac{\partial g}{\partial x} - D_p \frac{\partial^2 g}{\partial x^2} \right), \quad (81)$$

where

$$D = D_n^a + D_E + D_\xi, \mu = \mu_n^a + \mu_\xi, \quad (82)$$

$$D_E = \xi \times \tau_p \times \mu_p \times \mu_n \times E_0^2, \quad (83)$$

$$Q = \xi \times D_n \times D_p \times \tau_p, D_\xi = \xi_p \times \frac{\tau_p}{\tau_n} \times D_p + \xi_n \times D_n, \mu_\xi = \xi_p \times \frac{\tau_p}{\tau_n} \times \mu_p - \xi_n \times \mu_n. \quad (84)$$

Equation (81) is the desired equation. It depicts adequately the continuity of electron, hole, and total currents [see (15)-(17)].

## 7. Solitary Illuminated Sample

In this section, we will consider opportunities for improving photoexcitation of charge carriers and photo-emf  $V_{ph}$  by increasing concentration  $N$  of recombination centers.

### 7.1. Preliminaries: Basic relations

We will call the sample as solitary, if it is not in external electric field and external electrical circuit is open.

For simplicity, we will characterize effectiveness of charge carriers' photoexcitation by light-propagation-length averaged concentrations of photoelectrons  $\langle \Delta n \rangle$  and photoholes  $\langle \Delta p \rangle$  (Figure 5).

Figures 6 and 7 show calculated dependences  $\langle \Delta n \rangle(N)$  and  $V_{ph}(N)$ .

We have not used in study quasi-neutrality approximation [2-9, 13, 18, 21, 22, 28-31] because it can lead to unacceptable errors in calculation of dependences  $\langle \Delta n \rangle(N)$ ,  $\langle \Delta p \rangle(N)$  (Figures 8 and 9), and  $V_{ph}(N)$  (Figures 10 and 11) due to the fundamental contribution of photoexcited space charge into photoelectric effects in semiconductors. In other words, even in solitary sample, photoexcited electron-hole plasma in semiconductor may not always be quasi-neutral.

Let's consider a homogeneous semiconductor sample (Figure 5) with no voltage applied, i.e., in absence of illumination and intensity of electric field  $E_0=0$ . The density of photogeneration rate of charge carriers, in view of multiple internal reflections, is determined by the following expression:

$$g(x) = \gamma \times [a_- \times \exp(-\gamma x) + a_+ \times \exp(\gamma x)], \quad (85)$$

in which

$$a_- = \frac{(1-R) \times F_0}{1 - R^2 \times \exp(-2\gamma \tilde{W})}, a_+ = a_- \times R \times \exp(-2\gamma \tilde{W}), \quad (86)$$

where  $R$  and  $\gamma$  are coefficients of light reflection and absorption,  $F_0$  is density of incident photon flux, and  $\tilde{W}$  is sample thickness along incident light direction (Figure 5a). As is clear from (66) and (81), in discussed conditions, relation between concentrations of excess holes  $\Delta p$  and electrons  $\Delta n$  is as follows:

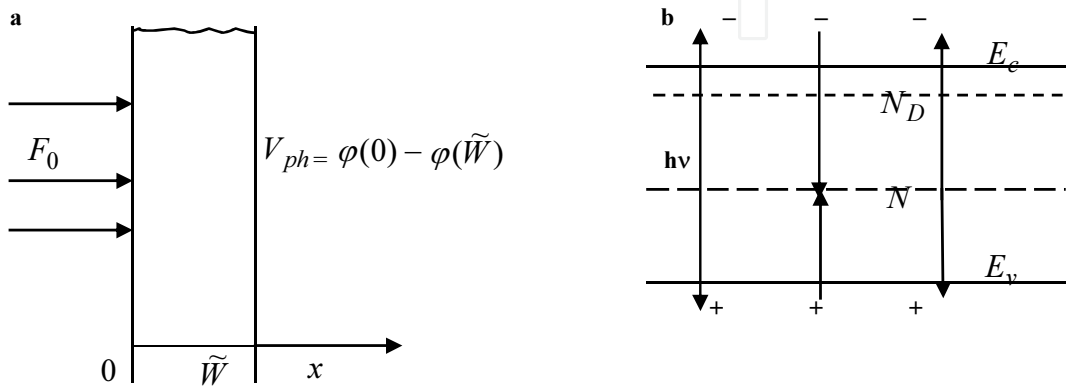
$$\Delta p = \frac{\tau_p}{\tau_n} \times \Delta n + \chi \times \left( \frac{\tau_p}{\tau_n} \times \Delta n - g \times \tau_p - D_n \times \tau_p \times \frac{\partial^2 \Delta n}{\partial x^2} \right), \quad (87)$$

and function  $\Delta n(x)$  obeys the equation

$$Q \times \frac{\partial^4 \Delta n}{\partial x^4} - D \times \frac{\partial^2 \Delta n}{\partial x^2} + \frac{\Delta n}{\tau_n} = g(x) - \xi \times \tau_p \times D_p \times \frac{\partial^2 g}{\partial x^2}, \quad (88)$$

where

$$D = D_n^a + D_\xi. \quad (89)$$



**Figure 5.** Layout of sample illumination (a) and generation-recombination processes in semiconductor (b).  $F_0$ , density of incident photon flux;  $V_{ph}$ , photo-emf;  $\phi(x)$ , electric potential;  $\tilde{W}$ , thickness of sample along light propagation;  $E_c$  and  $E_v$ , energy of conduction band bottom and valence band top

Denote:  $\tau_n$  and  $\tau_p$  are electron (24) and hole (25) lifetimes;  $D_{n,p}$  are their diffusion constants and values  $D_n^a$ ,  $D_\xi$  and  $Q$  and dimensionless parameters  $\chi$  and  $\xi \ll 1$  are determined by expressions (73), (84), (67), and (75), respectively.

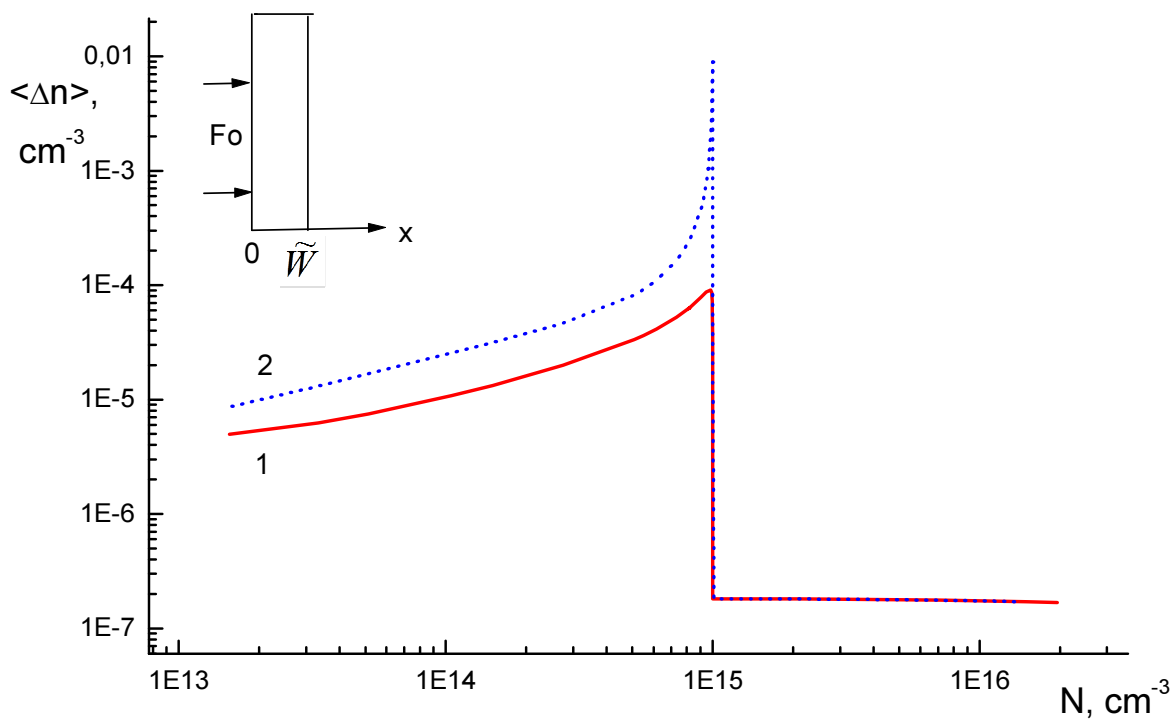
Exact solution of equation (88) is

$$\Delta n(x) = \sum_{i=1}^4 C_i \times \exp(k_i \times x) + T_n \times g(x), \quad (90)$$

where

$$k_{1,2} = \sqrt{\frac{L^2 - \sqrt{L^4 \mp 4 \times \xi \times L_n^2 \times L_p^2}}{2 \times \xi \times L_n^2 \times L_p^2}}, k_3 = -k_1, k_4 = -k_2, \quad (91)$$

$$T_n = \frac{\left( \frac{1}{\xi \times L_p^2} \right) - \gamma^2}{D_n \times (\gamma^2 - k_1^2) \times (\gamma^2 - k_2^2)}, \quad (92)$$



**Figure 6.** Dependences of mean concentration of photoelectrons  $\langle \Delta n \rangle$  in GaAs for levels with energy (eV):  $E_t = E_{t1} = -0.42$  (curve 1) and  $E_t = E_{t2} = -0.24$  (curve 2) on concentration of recombination impurity  $N$ ; layout of sample illumination and axis  $x$  are shown on insert. Adopted: light absorption coefficient  $\gamma = 10^4 \text{ cm}^{-1}$ , diffusion constants of electrons  $D_n = 221 \text{ cm}^2/\text{s}$  and holes  $D_p = 10.4 \text{ cm}^2/\text{s}$  [2-9];  $\tilde{W} = 10^{-3} \text{ cm}$ ;  $F_0 = 1 \text{ cm}^{-2}\text{s}^{-1}$ ;  $T = 300 \text{ K}$ ; concentration of shallow donors  $N_D = 10^{15} \text{ cm}^{-3}$ ;  $\theta \equiv w_p/w_n = 10^2$ ,  $w_n = 10^{-8} \text{ cm}^3/\text{s}$  [1-9], where  $w_p$  and  $w_n$  are capture probabilities of hole and electron

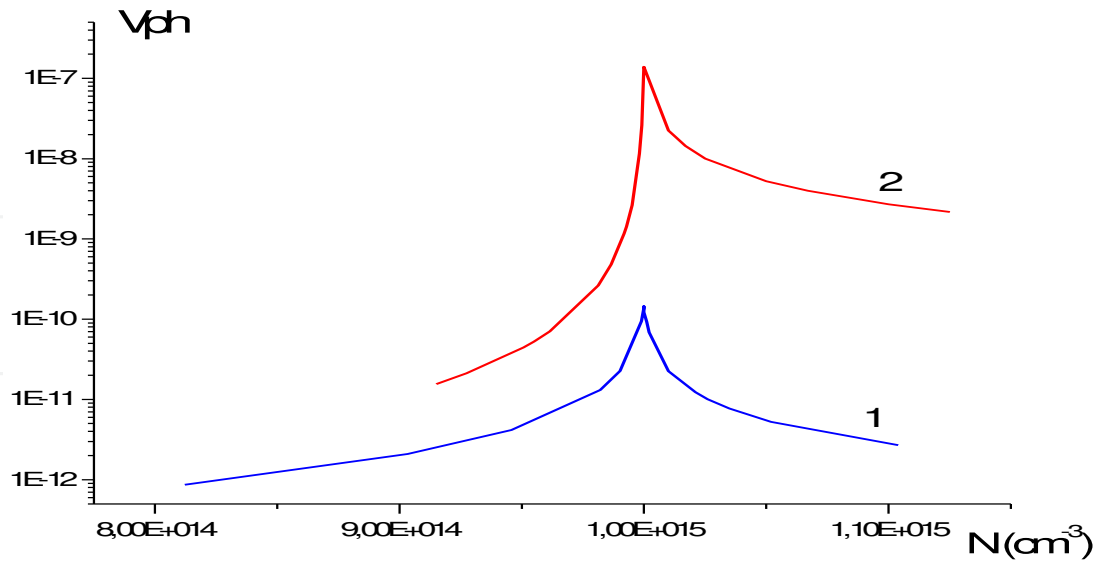
$$L^2 = L_a^2 + \xi_n \times L_n^2 + \xi_p \times L_p^2, \quad (93)$$

$$L_a^2 = \frac{p_e \times L_n^2 + b \times n_e \times L_p^2}{p + b \times n_e} = \frac{B \times \tau_n + \delta^2 \times \tau_p}{B + b \times \delta^2} \times D_n \equiv D_n^a \times \tau_n = D_p^a \times \tau_p, \quad (94)$$

$L_n = \sqrt{D_n \times \tau_n}$  and  $L_p = \sqrt{D_p \times \tau_p}$  are electron and hole diffusion lengths,  $D_p^a$  is quasi-neutral ambipolar hole diffusion constant, and  $L_a$  is quasi-neutral ambipolar diffusion length of charge carriers.

In quasi-neutrality approximation, parameters  $\xi$ ,  $\xi_n$  and  $\xi_p$  are equal to zero; therefore, in this approximation, the distribution of excess electrons' concentration is determined by equation

$$D_n^a \times \frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} = -g(x). \quad (95)$$



**Figure 7.** Dependence of photo-emf  $V_{ph}$  (arbitrary units) in  $GaAs$  for levels with energy  $E_{t1}$  (curve 1) and  $E_{t2}$  (curve 2) on concentration of recombination impurity  $N$ . Parameters are the same as in Figure 6

## 7.2. Effectiveness of charge carriers' photoexcitation

We define the mean value  $\langle y \rangle$  of variable  $y(x)$  as

$$\langle y \rangle = \frac{1}{\tilde{W}} \int_0^{\tilde{W}} y(x) dx. \quad (96)$$

Let's analyze the worst case, when recombination of excess charge carriers on illuminated ( $x=0$ ) and unilluminated ( $x=\tilde{W}$ ) surfaces of the sample (Figure 5a) is so intensive that

$$\Delta n(0) = \Delta p(0) = \Delta n(\tilde{W}) = \Delta p(\tilde{W}) = 0. \quad (97)$$

From (87), (90), and (97), it follows that

$$\Delta n(x) = \Delta n_1(x) + \Delta n_2(x) + T_n \times g(x), \quad (98)$$

$$\Delta n_1(x) = \frac{k_1^2 \times (L_n^2 \times k_2^2 - 1)}{D_n \times (k_2^2 - k_1^2) \times (\gamma^2 - K_1^2)} \times \frac{g(0) \times sh[k_1 \times (\tilde{W} - x)] + g(\tilde{W}) \times sh(k_1 \times x)}{sh(k_1 \times \tilde{W})}, \quad (99)$$

$$\Delta n_2(x) = -\frac{k_2^2 \times (L_n^2 \times k_1^2 - 1)}{D_n \times (k_2^2 - k_1^2) \times (\gamma^2 - k_2^2)} \times \frac{g(0) \times sh[k_2 \times (\tilde{W} - x)] + g(\tilde{W}) \times sh(k_2 \times x)}{sh(k_2 \times \tilde{W})}, \quad (100)$$



$$\Delta p(x) = \frac{\tau_p}{\tau_n} \times \Delta n(x) - \frac{\tau_p}{\tau_n} \times \chi \times \frac{(L_n^2 \times k_1^2 - 1) \times (L_n^2 \times k_2^2 - 1)}{D_n} \times \left\{ \frac{A_1 + A_2}{k_2^2 - k_1^2} + \frac{\gamma^2 \times g(x)}{(\gamma^2 - k_1^2) \times (\gamma^2 - k_2^2)} \right\} \quad (101)$$

where

$$A_1 = \frac{k_1^2}{\gamma^2 - k_1^2} \times \frac{g(o) \times sh[k_1 \times (\tilde{W} - x)] + g(\tilde{W}) \times sh(k_1 \times x)}{sh(k_1 \times \tilde{W})}, \quad (102)$$

$$A_2 = -\frac{k_2^2}{\gamma^2 - k_2^2} \times \frac{g(o) \times sh[k_2 \times (\tilde{W} - x)] + g(\tilde{W}) \times sh(k_2 \times x)}{sh(k_2 \times \tilde{W})}. \quad (103)$$

Thus, according to definition (96), we find

$$\langle \Delta n \rangle = \eta_n \times \langle g \rangle, \langle \Delta p \rangle = \eta_p \times \langle g \rangle, \quad (104)$$

where

$$\eta_n = T_n + \frac{\gamma \times cth(\gamma \times \tilde{W} / 2)}{(k_2^2 - k_1^2) \times D_n} \times \sum_{i=1}^2 (-1)^{i-1} \times \frac{k_i \times \alpha_{3-i}}{\gamma^2 - k_i^2} \times th\left(\frac{k_i \times \tilde{W}}{2}\right), \quad (105)$$

$$\alpha_{1,2} = k_{1,2}^2 \times L_n^2 - 1, \quad (106)$$

$$\langle g \rangle = \frac{(1-R) \times F_0}{W} \times \frac{1 - \exp(-\gamma \times \tilde{W})}{1 - R \times \exp(-\gamma \times \tilde{W})}, \quad (107)$$

$$\eta_p = \frac{4\pi}{\varepsilon} \times (a_n + a_p) \times \tau_p \times \eta_p + \frac{\tau_p}{\tau_n} \times \eta_n, \quad (108)$$

$$\eta_p = -\frac{\varepsilon}{4\pi} \times \frac{\gamma \times \alpha_1 \times \alpha_2}{(\mu_n \times n_e - a_n) \times L_n^2} \times \left\{ \Lambda^3 - \frac{cth(\gamma \times \tilde{W} / 2)}{k_2^2 - k_1^2} \times \sum_{i=1}^2 (-1)^i \times \frac{k_i}{\gamma^2 - k_i^2} \times th\left(\frac{k_i \times \tilde{W}}{2}\right) \right\} \equiv \frac{\langle \Delta \rho \rangle}{\langle g \rangle}, \quad (109)$$

$$\Lambda^3 = \frac{\gamma}{(\gamma^2 - k_1^2)(\gamma^2 - k_2^2)}, \quad (110)$$

and  $\Delta\rho$  is photoexcited space charge density.

Outside quasi-neutrality approximation, expressions (20)–(25), (60), (61), (75), (80), (91)–(94), and (104)–(110) determine, in parametric form (value  $\delta = N_-^e / N_0^e$  is used as parameter), dependences  $\langle \Delta n \rangle(N)$  and  $\langle \Delta p \rangle(N)$ . We will call found dependences (see Figures 6, 8, and 9) as exact, because, in linear approximation with respect to  $F_0$ , dependences are exact in contrast to quasi-neutrality approximation case.

Denote desired dependences in quasi-neutrality approximation as  $\langle \Delta \tilde{n}(N) \rangle$  and  $\langle \Delta \tilde{p}(N) \rangle$ . In quasi-neutrality approximation,

$$\Delta n(x) = \Delta \tilde{n} \equiv \frac{\tau_n}{1 - \gamma^2 \times L_a^2} \times \left\{ g(x) - \frac{g(0) \times sh[(\tilde{W} - x) / L_a] + g(\tilde{W}) \times sh(x / L_a)}{sh(\tilde{W} / L_a)} \right\}, \quad (111)$$

$$\langle \Delta p \rangle = \langle \Delta \tilde{p} \rangle \equiv \langle \Delta \tilde{n} \rangle \times \frac{\tau_p}{\tau_n}, \quad (112)$$

$$\langle \Delta n \rangle = \langle \Delta \tilde{n} \rangle \equiv \frac{(1 - R) \times F_0 \times \tau_n}{(1 - \gamma^2 \times L_a^2) \times \tilde{W}} \times \frac{1 - \exp(-\gamma \tilde{W}) - \gamma \times L_a \times [1 + \exp(-\gamma \tilde{W})] \times th(\tilde{W} / 2L_a)}{1 - R \times \exp(-\gamma \tilde{W})}. \quad (113)$$

Figure 6 shows that the effectiveness of charge carriers' photoexcitation may grow significantly with increasing  $N$ . Up to small corrections, dependences  $\langle \Delta n \rangle(N)$  and  $\langle \Delta p \rangle(N)$  reach maximums  $\langle \Delta n \rangle_{\max}$  and  $\langle \Delta p \rangle_{\max}$  at the same concentration  $N = \tilde{N} = N_D$  as for lifetimes (Figure 1b) and after that fall very strongly. Figures 8 and 9 illustrate the influence of photoexcited space charge in point  $N = \tilde{N}$  on the validity of results. It is clear from Figures 8 and 9 that with thinning  $\tilde{W}$  of sample, using quasi-neutrality approximation leads to error up to several orders of magnitude. Let's clarify Figures 6, 8, and 9.

Lengths  $L$  and  $L_1 \equiv 1/k_1$ , up to small corrections, equal to  $L_a$ ; moreover

$$L_n > L_1 > L_2 \equiv \frac{1}{k_2}. \quad (114)$$

At  $\tilde{W} \ll L_a$ , we have

$$\langle \Delta \tilde{n} \rangle = \frac{\psi(\tilde{W}, \gamma)}{D_n^a}, \langle \Delta \tilde{p} \rangle = \frac{\psi(\tilde{W}, \gamma)}{D_p^a}, \quad (115)$$

where function  $\psi(\tilde{W}, \gamma)$  is independent on  $\tau_n$  and  $\tau_p$ . On the other hand, if inequality  $\tilde{W} < L_2$  is sufficiently strong, and when  $L_p > \tilde{W}$  as well, from expressions (104), (105), and (108)–(110), it follows that

$$\langle \Delta n \rangle = \frac{\psi(\tilde{W}, \gamma)}{D_n}, \langle \Delta p \rangle = \frac{\psi(\tilde{W}, \gamma)}{D_p}. \quad (116)$$

This means that diffusion of photoelectrons and photoholes is independent from each other.

Therefore,  $L_2$  has physical meaning as shielding length of photoinduced space charge.

Length  $L_2 < L_a$ ,  $\tilde{W}$  in the vicinity of the point  $N = \hat{N}$ , i.e. quasi-neutrality is valid. At values  $N < \hat{N}$ , length  $L_a < \tilde{W}$  due to small value  $D_n^a$ . When  $N$  passing through the point  $\hat{N}$  toward larger values  $N$ , then  $D_n^a$  increases very strongly (by several orders of magnitude), and after that it is remaining substantially constant. As a result, length  $L_a > \tilde{W}$  at values  $N$  higher  $\hat{N}$ . This explains the asymmetry of dependence  $\langle \Delta n \rangle(N)$  about point  $N = \hat{N}$ , and “plateau,” when  $N > \hat{N}$  as well (Figure 6).

At point  $N = \hat{N}$  and about it, length  $L_2$  is so long that even at  $\tilde{W} \sim 0.1$  cm, solution in quasi-neutrality approximation is unacceptable; moreover, with decreasing  $\tilde{W}$

$$\frac{\langle \Delta n \rangle_{\max}}{\langle \Delta \tilde{n} \rangle_{\max}} \rightarrow \frac{D_n^a}{D_n} \ll 1, \quad (117)$$

and

$$\frac{\langle \Delta p \rangle_{\max}}{\langle \Delta n \rangle_{\max}} \rightarrow \frac{D_n}{D_p} \cong 20, \quad (118)$$

despite the fact that  $\tau_p < \tau_n$  (Figure 1b).

When  $\gamma^{-1} < \tilde{W}$  and inequity  $\tilde{W} < L_n$  are sufficiently strong, then the total quantity of photoelectrons is proportional to  $\tilde{W}$  due to reducing loss on unilluminated surface. Therefore, there is a “plateau” on dependence  $\langle \Delta n \rangle_{\max}$  on  $\tilde{W}$ . For the same reason, at  $\gamma^{-1} < \tilde{W} < L_a$ , dependence  $\langle \Delta \tilde{n} \rangle_{\max}$  on  $\tilde{W}$  has a “plateau” as well. However, in the last case, “plateau” height is much higher, and its width is much wider than true “plateau” (Figure 8, curve 1).

Moreover, shielding length of photoinduced space charge  $L_2 \approx L_a$  when impurity level energy equals to  $E_{t2}$ . Therefore, solution obtained in quasi-neutrality approximation, even when  $\tilde{W} \rightarrow \infty$ , differs from exact solution at least by several times (Figures 8 and 9, curve 2).

### 7.3. Effectiveness of photo-emf excitation

In view of the fact that under considered conditions

$$I_{ph} = 0, \quad (119)$$

from expressions (12)–(14) and (87), we obtain that photo-emf

$$V_{ph} = \int_0^{\tilde{W}} \Delta E(x) dx = \phi(0) - \phi(\tilde{W}), \quad (120)$$

i.e. potential  $\phi(x)$  of illuminated surface with respect to unilluminated (Figure 5a) expressed by formula

$$V_{ph} = \frac{D_n \times \Delta_n - D_p \times \Delta_p}{\mu_n \times n_e + \mu_p \times n_e} = \frac{(L_n^2 - L_p^2) \times \Delta_n + \chi \times L_p^2 \times (\tau_n \times \Delta_g + L_n^2 \times \Delta_n'' - \Delta_n)}{(\mu_n \times n_e + \mu_p \times n_e) \times \tau_n}, \quad (121)$$

where

$$\Delta_n = \Delta n(0) - \Delta n(\tilde{W}), \Delta_p = \Delta p(0) - \Delta p(\tilde{W}), \quad (122)$$

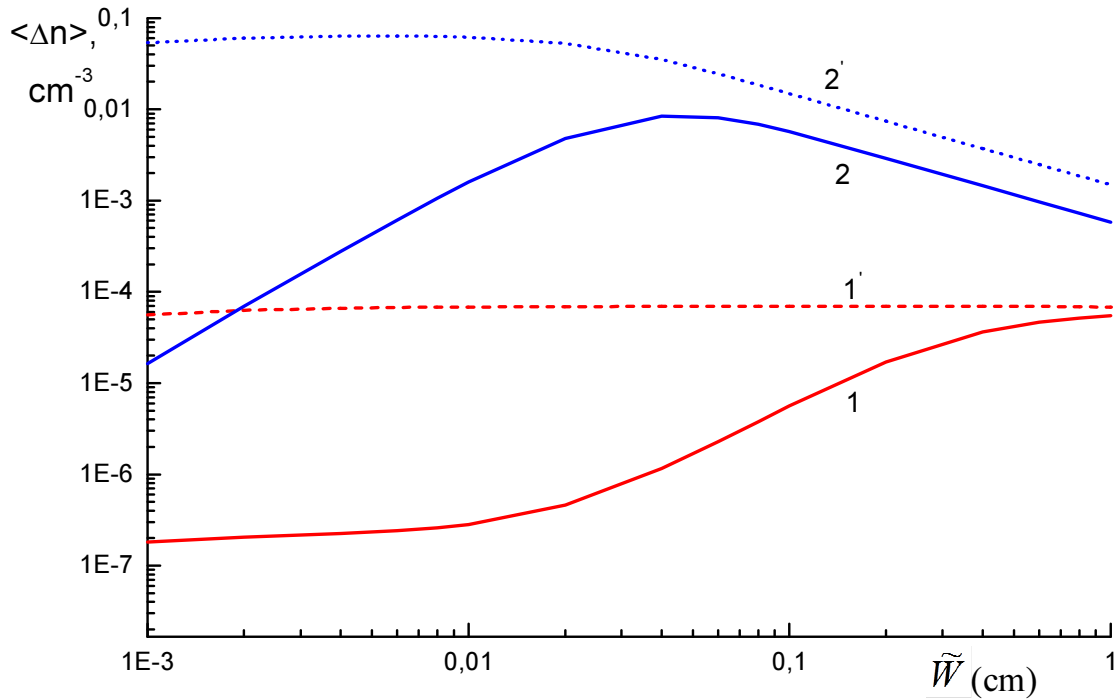
$$\Delta_g = g(0) - g(\tilde{W}), \Delta_n'' = \left. \frac{\partial^2 \Delta n}{\partial x^2} \right|_{x=0} - \left. \frac{\partial^2 \Delta n}{\partial x^2} \right|_{x=\tilde{W}}. \quad (123)$$

It is clear from expression (121) that at infinite surface recombination rate, i.e., when conditions (97) are fulfilled, illumination produces no photo-emf. In this regard, assume that photoexcited charge carriers are not captured on surfaces and there is no charge on surfaces. In these conditions, photoexcited electric field intensity

$$\Delta E(0) = \Delta E(\tilde{W}) = 0 \quad (124)$$

And densities of electron  $\Delta I_n$  and hole  $\Delta I_p$  photocurrents

$$\Delta I_n(0) = \Delta I_p(0) = \Delta I_n(\tilde{W}) = \Delta I_p(\tilde{W}) = 0. \quad (125)$$



**Figure 8.** Dependences of  $\langle \Delta n \rangle_{\max}$  in the point of maximum functions  $\langle \Delta n \rangle(N)$  and  $\langle \Delta p \rangle(N)$  on thickness  $\tilde{W}$  for *GaAs*. Curves 1 and 2, recombination level energy  $E_t$  equals to  $E_{t1}$  and  $E_{t2}$ , respectively; solid lines - exact solutions; dashed lines - solutions in approximation of quasi-neutrality. Adopted parameters and other symbols are the same as in Figure 6

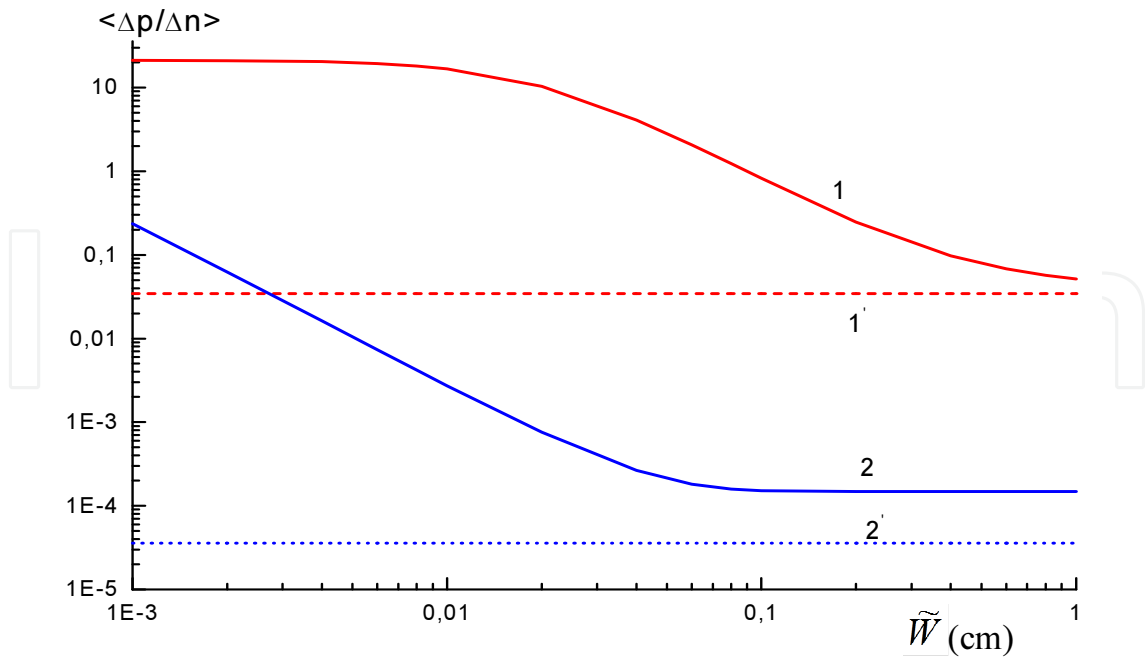
Therefore, we may write boundary conditions, in view of relation (87), as

$$\left. \frac{\partial \Delta n}{\partial x} \right|_{x=0} = \left. \frac{\partial \Delta n}{\partial x} \right|_{x=\tilde{W}} = 0, \quad (126)$$

$$D_n \times \left. \frac{\partial^3 \Delta n}{\partial x^3} \right|_{x=0} = - \left. \frac{\partial g}{\partial x} \right|_{x=0}, D_n \times \left. \frac{\partial^3 \Delta n}{\partial x^3} \right|_{x=\tilde{W}} = - \left. \frac{\partial g}{\partial x} \right|_{x=\tilde{W}}. \quad (127)$$

From (85), (86), (90), (126), and (127), it follows that

$$C_{1,2} = (-1)^{2,1} \times b_{1,2} \times \frac{g'_0 \times \exp(-k_{1,2} \times \tilde{W}) - g'_{\tilde{W}}}{2 \times sh(k_{1,2} \times \tilde{W})}, \quad (128)$$



**Figure 9.** Dependences of ratio  $\langle \Delta p \rangle_{\max} / \langle \Delta n \rangle_{\max}$  in the point of maximum functions  $\langle \Delta n \rangle(N)$  and  $\langle \Delta p \rangle(N)$  on thickness  $\tilde{W}$  for GaAs. Curves 1 and 2, recombination level energy  $E_t$  equals to  $E_{t1}$  and  $E_{t2}$ , respectively; solid lines - exact solutions; dashed lines - solutions in approximation of quasi-neutrality. Adopted parameters and other symbols are the same as in Figure 6

$$C_{3,4} = C_{1,2} \times \frac{g'_0 \times \exp(k_{1,2} \times \tilde{W}) - g'_{\tilde{W}}}{g'_0 \times \exp(-k_{1,2} \times \tilde{W}) - g'_{\tilde{W}}}, \quad (129)$$

where

$$b_{1,2} = \frac{k_{1,2} \times (k_{2,1}^2 \times L_n^2 - 1)}{D_n \times (k_2^2 - k_1^2) \times (k_{1,2}^2 - \gamma^2)}, \quad (130)$$

$$g'_0 \equiv \left. \frac{\partial g}{\partial x} \right|_{x=0} = -\gamma^2 \times a_- \times [1 - R \times \exp(-2\gamma\tilde{W})], \quad (131)$$

$$g'_{\tilde{W}} \equiv \left. \frac{\partial g}{\partial x} \right|_{x=\tilde{W}} = \frac{(1 - R) \times \exp(-\gamma\tilde{W})}{1 - R \times \exp(-2\gamma\tilde{W})} \times g'_0. \quad (132)$$

Find consistently

$$\Delta n(0) = T_n \times g_0 + \sum_{i=1}^2 (-1)^{i+1} b_i \times [g'_0 \times \text{cth}(k_i \times \tilde{W}) - g'_{\tilde{W}} \times \text{cosech}(k_i \times \tilde{W})], \quad (133)$$

$$\Delta n(\tilde{W}) = T_n \times g_{\tilde{W}} + \sum_{i=1}^2 (-1)^{i+1} b_i \times [g'_0 \times \text{cosech}(k_i \times \tilde{W}) - g'_{\tilde{W}} \times \text{cth}(k_i \times \tilde{W})], \quad (134)$$

where

$$g_0 \equiv g(0) = \gamma \times a_- \times [1 + R \times \exp(-2\gamma\tilde{W})], g_{\tilde{W}} \equiv g(\tilde{W}) = \frac{(1+R) \times \exp(-\gamma\tilde{W})}{1+R \times \exp(-2\gamma\tilde{W})} \times g_0. \quad (135)$$

$$\Delta_n = T_n \times \Delta_g + \sum_{i=1}^2 (-1)^{i+1} \times b_i \times \text{th}\left(\frac{k_i \times \tilde{W}}{2}\right) \times \Sigma'_g, \quad (136)$$

$$\Delta_n'' = \gamma^2 \times T_n \times \Delta_g + \sum_{i=1}^2 (-1)^{i+1} \times k_i^2 \times b_i \times \text{th}\left(\frac{k_i \times \tilde{W}}{2}\right) \times \Sigma'_g, \quad (137)$$

where

$$\Delta_g = g(0) - g(\tilde{W}) = \gamma \times F_0 \times (1-R) \times \frac{1 - \exp(-\gamma\tilde{W})}{1 + R \times \exp(-\gamma\tilde{W})}, \quad (138)$$

$$\Sigma'_g = g'_0 + g'_{\tilde{W}} = -\gamma^2 \times F_0 \times (1-R) \times \frac{1 + \exp(-\gamma\tilde{W})}{1 + R \times \exp(-\gamma\tilde{W})}. \quad (139)$$

If we utilize relation

$$Z \equiv L_n^2 - L_p^2 - \chi \times L_p^2 \times (k_1^2 \times L_n^2 - 1) \times (k_2^2 \times L_n^2 - 1) = 0, \quad (140)$$

then expressions (121) and (136)–(139) allow us to obtain

$$V_{ph} = \frac{(1-R) \times \gamma \times F_0 \times (L_n^2 - L_p^2)}{\mu_n \times n_e + \mu_p \times p_e} \times M, \quad (141)$$



where

$$M = \frac{k_1^2 \times k_2^2 \times f_-(\gamma \tilde{W})}{(k_1^2 - \gamma^2) \times (k_2^2 - \gamma^2)} + \frac{\gamma \times k_1 \times k_2}{k_2^2 - k_1^2} \times \left[ \frac{k_1 \times th(k_2 \times \tilde{W} / 2)}{k_2^2 - \gamma^2} - \frac{k_2 \times th(k_1 \times \tilde{W} / 2)}{k_1^2 - \gamma^2} \right] \times f_+(\gamma \tilde{W}), \quad (142)$$

$$f_{-,+}(\gamma \tilde{W}) = \frac{1 \mp \exp(-\gamma \tilde{W})}{1 + R \times \exp(-\gamma \tilde{W})}. \quad (143)$$

In quasi-neutrality approximation, we get

$$V_{ph} = \tilde{V} \equiv \frac{(1 - R) \times \gamma \times F_0 \times (L_n^2 - L_p^2)}{\mu_n \times n_e + \mu_p \times p_e} \times \tilde{M}, \quad (144)$$

where

$$\tilde{M} = \frac{\gamma \times L_a \times f_+(\gamma \tilde{W}) \times th(\tilde{W} / 2L_a) - f_-(\gamma \tilde{W})}{(\gamma \times L_a)^2 - 1}. \quad (145)$$

We refer to the dependences (141) of photo-emf  $V_{ph}$  on  $N$  and  $\tilde{W}$  as exact. The reason is that, in contrast to the case of quasi-neutrality approximation, said dependences are exact in linear approximation with respect to flux density  $F_0$ .

Figures 7, 10, and 11 show these dependences and solution in quasi-neutrality approximation as well. It is clear in Figures 10 and 11 that, in quasi-neutrality approximation, maximal value  $V_{ph}(N)$  far exceeds the “true” value; moreover, exceedance may be several orders of magnitude.

We can explain results by the fact that very long shielding length (114) of photoexcited space charge  $L_2$  in the point of maximum  $N = \hat{N} \equiv N_D$  and nearby will cause diffusion of photoelectrons independently on photoholes (see Section 6.2) at  $\tilde{W} \sim 0.1$  cm. In other words, electron diffusion constant  $D_n$  determines the distribution of photoelectron concentration  $\Delta n(x)$ . Aside from that, at  $N \cong \hat{N}$ , due to charge coupled to impurities, inequity  $D_n \gg D_n^a$  is fulfilled. It means that true effectiveness of photoelectrons' spreading is much higher, than that given by quasi-neutrality approximation. Therefore, ratio  $(V_{ph})_{\max} / (\tilde{V}_{ph})_{\max} \ll 1$ , when  $\tilde{W} \gg L_2$  (Figure 11).

Evidently, with thinning  $\tilde{W}$ , spreading area of photoexcited charge carriers becomes wider, i.e., values  $\Delta_n = \Delta n(0) - \Delta n(\tilde{W})$  и  $\Delta_p = \Delta p(0) - \Delta p(\tilde{W})$  decrease. Therefore, values  $(V_{ph})_{\max}$  and  $(\tilde{V}_{ph})_{\max}$  fall with thinning  $\tilde{W}$  (Figure 10).

Note that for impurity level energy equal to  $E_{t2}$ , the solution obtained in quasi-neutrality approximation, even when  $\bar{W} \rightarrow \infty$ , differs from the exact solution more than two times (Figure 10). This is due to the fact that in considered case the shielding length of photoexcited space charge  $L_2 = 1/k_2 \approx L_a$ , where  $L_a$  is ambipolar diffusion length of charge carriers.

## 8. Photoelectric gain

We will consider uniform spatial distribution of density of charge carriers' photoexcitation rate  $g$  and ignore surface recombination of photoexcited charge carriers.

We will facilitate mathematical description of photoelectric gain (see Figures 1a and 2)

$$G(N, V) \equiv \frac{I_{ph}}{q \times W \times g} \quad (146)$$

through utilizing small dimensionless parameter (75)  $\xi < 1$ , which will characterize the degree of deviation of semiconductor from local neutrality under illumination. Here  $N$  is concentration of recombination centers;  $I_{ph}$  is photocurrent density (14);  $q$  is absolute value of electron charge;  $V$  is bias voltage applied across the sample;  $W$  is distance between current electrodes (see insert in Figure 1a).

Using linearized expressions for electron (12) and hole (13) components of photocurrent  $I_{ph}$  and expressions (60) and (61), we may rewrite equations (5)–(7) as follows:

$$R_n = \frac{\Delta n}{\tau_n} - \xi_n \times \frac{\partial j}{\partial x}, R_p = \frac{\Delta p}{\tau_p} + \xi_p \times \frac{\partial j}{\partial x}, \Delta p = \frac{\tau_p}{\tau_n} \times \Delta n - \xi \times \tau_p \times \frac{\partial j}{\partial x}, \quad (147)$$

where

$$j = (\mu_n \times \Delta n + \mu_p \times \Delta p) \times E_0 + D_n \times \frac{\partial \Delta n}{\partial x} - D_p \times \frac{\partial \Delta p}{\partial x}, \quad (148)$$

$E_0 = V / W$ , and dimensionless small parameters  $\xi_n < 1$  and  $\xi_p < 1$  as defined in (80). Relations (147) show that, by dimensionless small parameter  $\xi$ , we may really characterize the degree of deviation of semiconductor from local neutrality under illumination. If there is no external load (inset in Figure 1a), then illumination does not change voltage  $V$  across the sample.

Therefore, expressions (4), (5), (8), and (9) with boundary conditions (1) allow us to write

$$I_{ph} = \left\{ \left( b + \frac{\tau_p}{\tau_n} \right) \times \langle \Delta n \rangle + \xi \times \frac{L_p^2}{W} \times \left( \frac{\partial \Delta p}{\partial x} - b \times \frac{\partial \Delta n}{\partial x} \right) \right\}_{x=0}^{x=W} \times q \times \mu_p \times E_0, \quad (149)$$

where  $\langle n \rangle$  is arithmetic mean with respect to  $x$  concentration of excess electrons (insert in Figure 1a)

$$\langle \Delta n \rangle = \frac{1}{W} \times \int_0^W \Delta n(x) dx, \quad (150)$$

$$b = \mu_n / \mu_p.$$

Actual distribution  $\Delta n(x)$  at  $g(x) = \text{const}$  is defined by equation

$$Q \times \frac{\partial^4 \Delta n}{\partial x^4} - (D_n^a + D_E + D_\xi) \times \frac{\partial^2 \Delta n}{\partial x^2} + (\mu_n^a + \mu_\xi) \times E_0 \times \frac{\partial \Delta n}{\partial x} + \frac{\Delta n}{\tau_n} = g. \quad (151)$$

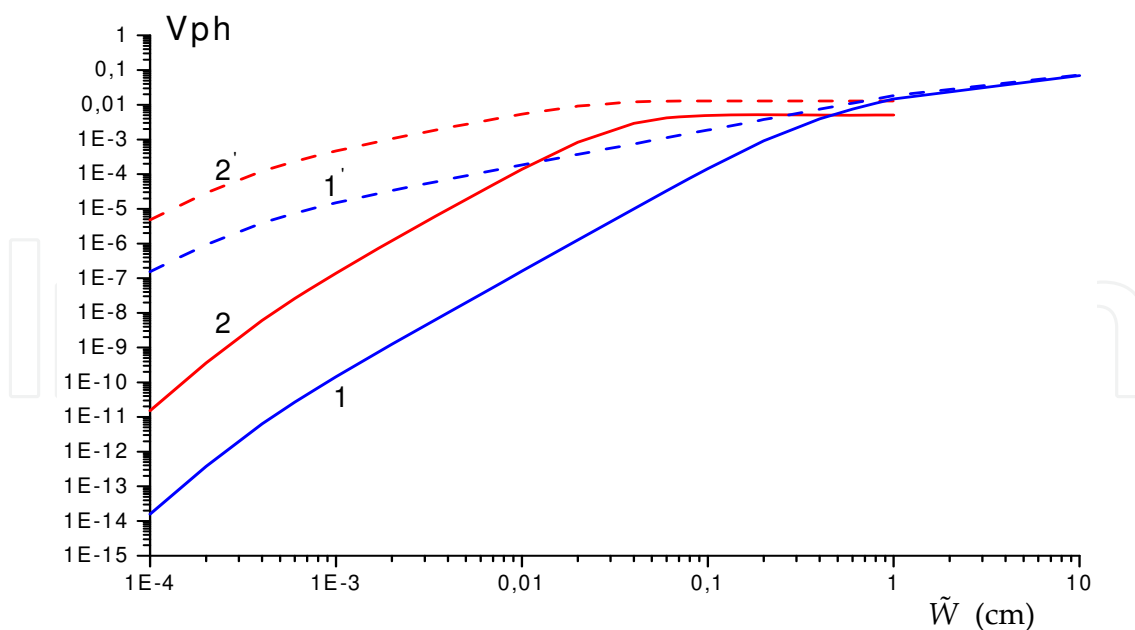
Values of parameters  $Q$ ,  $D_\xi$ ,  $\mu_\xi$ , and  $D_E$  are dictated by photoexcitation of space charge  $\Delta \rho$  (4). Analysis shows that inequality  $\xi < 1$ , which is typically fulfilled with large margin, allowing to omit in equation (151) terms with parameters  $Q$ ,  $D_\xi$  and  $\mu_\xi$ . However, term with parameter  $D_E$  must be retained, because, even at moderate electric fields,  $D_E$  may exceed  $D_n^a$  due to square-law dependence  $D_E$  on  $E_0$ . For the same reason, we can omit in equation (149) terms including small parameter  $\xi$ . Thus, we arrive at relations

$$(D_n^a + D_E) \times \frac{\partial^2 \Delta n}{\partial x^2} - \mu_n^a \times E_0 \times \frac{\partial \Delta n}{\partial x} - \frac{\Delta n}{\tau_n} + g = 0, \quad (152)$$

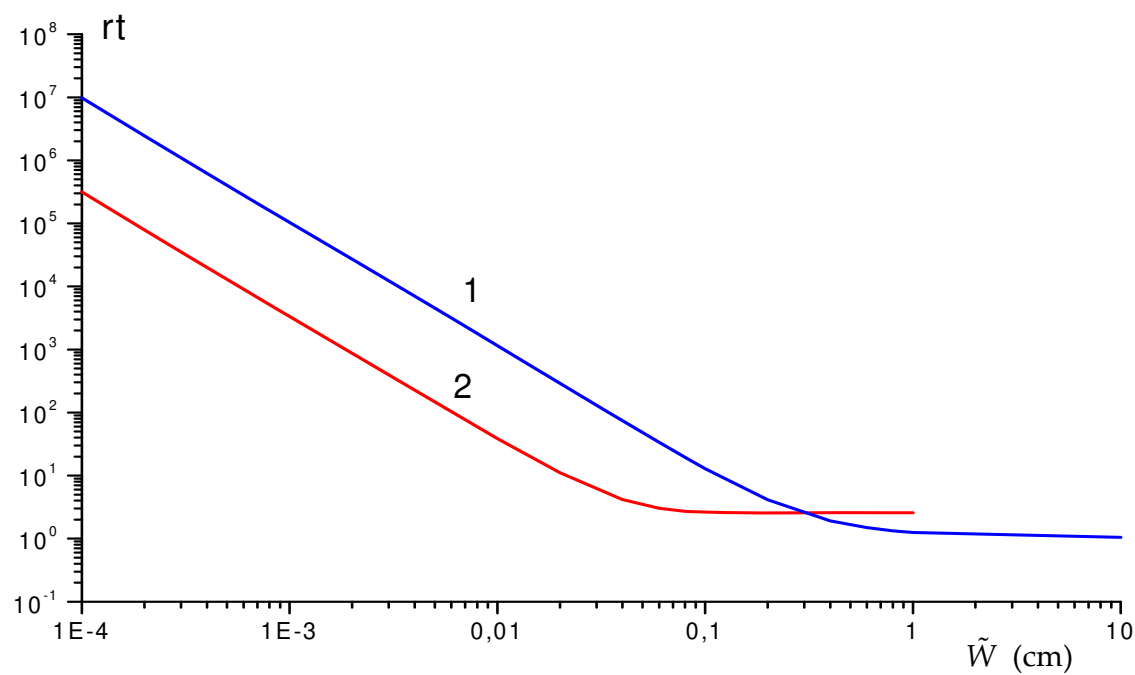
$$I_{ph} = q \times \left( \mu_n + \frac{\tau_p}{\tau_n} \times \mu_p \right) \times \langle \Delta n \rangle \times E_0. \quad (153)$$

Equation (152), with boundary condition (1), and relation (153) allow us to obtain the formula for photoelectric gain:

$$G = \left( \mu_n \times \tau_n + \mu_p \times \tau_p \right) \times \frac{E_0}{W} \times \left\{ 1 - 4 \times \frac{L_{ef}}{W} \times \sqrt{\left( \frac{d_a}{2L_{ef}} \right)^2 + 1} \times \frac{\text{sh} \left( \frac{W}{2L_1} \right) \times \text{sh} \left( \frac{W}{2L_2} \right)}{\text{sh} \left( \frac{W}{2L_1} + \frac{W}{2L_2} \right)} \right\}, \quad (154)$$



**Figure 10.** Dependences of maximal value  $V_{ph}(N)$  in *GaAs* on thickness  $\tilde{W}$ . Curves 1 and 2, recombination level energy  $E_t$  equals to  $E_{t1}$  and  $E_{t2}$ , respectively; solid curves, exact solutions; dashed curves, solutions in approximation of quasi-neutrality. Adopted parameters and other symbols are the same as in Figure 6



**Figure 11.** Dependence of ratio  $rt = \tilde{V}_{ph} / V_{ph}$  in the point of maximal value  $V_{ph}(N)$  in *GaAs* on thickness  $\tilde{W}$ . Curves 1 and 2 – recombination level energy  $E_t$  equals to  $E_{t1}$  and  $E_{t2}$ .  $V_{ph}$ , exact solutions;  $\tilde{V}_{ph}$ , solutions in approximation of quasi-neutrality. Adopted parameters and other symbols are the same as in Figure 6

where quasi-neutral ambipolar drift length of charge carriers

$$d_a = \mu_n^a \times \tau_n \times E_0, \quad (155)$$

effective diffusion length of charge carriers

$$L_{ef} = \sqrt{(D_n^a + D_E) \times \tau_n}, \quad (156)$$

and effective reciprocal diffusion-drift lengths  $L_1$  and  $L_2$  are defined by expressions

$$\frac{1}{L_{1,2}} = \pm \frac{d_a}{2L_{ef}^2} + \sqrt{\left(\frac{d_a}{2L_{ef}^2}\right)^2 + \frac{1}{L_{ef}^2}}. \quad (157)$$

Relations (18), (20)–(22), (24), (25), (73)–(75), (83), and (153)–(157) determine, in parametric form dependence  $G(N, V)$  (see Figures 1a and 2a). Ratio  $\delta = N_-^e / N_0^e$  is used as parameter in said relations.

It can be shown that equation  $\mu_n^a(N)=0$ , where  $\mu_n^a$  is given by expression (74), has a solution when inequalities (27) and (41) are fulfilled. In zeroth-order with respect to small parameters (27) and (41), the root of this equation coincides with maximal extrema of functions  $\tau_n(N)$  and  $\tau_p(N)$  and equals to  $N_D$  (Figure 1b and 1c).

Let's explain dependence  $\mu_n^a$  on  $N$  shown in Figure 1c.

Product

$$\mu_n^a \times \tau_n = \mu_p^a \times \tau_p \quad (158)$$

determines drift length and direction in electric field of concentrational perturbation – quasi-neutral cloud of positive and negative charges [2, 31], including bound at deep impurity (here  $\mu_n^a$  and  $\mu_p^a$  are electron and hole ambipolar mobility). Last mentioned bounding explains dependence  $\mu_n^a$  on ratio  $\tau_p(N) / \tau_n(N)$  in trap-assisted recombination. It would appear reasonable that charge carriers, which prevail in quantity, can easily shield photoexcited space charge, i.e., they are adjusted to drift of charge carriers of another type. That is why, in the case of band-to-band recombination ( $\tau_n = \tau_p$ ), quasi-neutral cloud of positive and negative charges drifts in electric field with the same velocity and in the same direction as minority charge carriers, whereas in intrinsic material, cloud is out of control by electric field at all [2, 31] ( $\mu_n^a = \mu_p^a \cong \mu_p > 0$  at  $n_e > p_e$ ,  $\mu_n^a = \mu_p^a \cong -\mu_n < 0$  at  $n_e < p_e$  and  $\mu_n^a = \mu_p^a = 0$  at  $n_e = p_e$ ). Similar situation, but not exactly identical, happens in the case of trap-assisted recombination.

In this case, due to the fact that  $\tau_p < \tau_n$  (Figure 1c), vanishing  $\mu_n^a$  occurs in n-type material (for specified parameters in Figure 1, at  $n_e \cong 10 \times p_e$  in silicon and  $n_e \cong 10^4 \times p_e$  in gallium arsenide). Positive sign of perturbation charge bound at deep impurities ( $\Delta N_- < 0$ ) causes such behavior. Ratio  $p_e/n_e$  begins to increase significantly, and very sharply, only when  $N \cong N_D$ . At the same time, ratio  $\tau_p/\tau_n$  may not have so many orders of smallness as ratio  $p_e/n_e$  may have. Therefore,  $\mu_n^a$  vanishes when  $N \cong N_D$ , if deep level, according to conditions (27) and (41), lies in lower half of bandgap. If that level lies in upper half of bandgap, then, again, due to the fact that  $\tau_p/\tau_n$  may not have so many orders of smallness as ratio  $p_e/n_e$  may have, function  $\mu_n^a(N)$  is always positive for actual values  $N$ , wherein lifetimes  $\tau_p$  and  $\tau_n$  decrease always with increasing  $N$  (see Section 3). We denote solution based on relations (152) and (153) as approximate. Parameter  $\xi(N)$ , still remaining small, reaches its global maximum near point  $N = \hat{N} = N_D$ , where function  $G(N)$  reaches maximal extremum  $\hat{G}$  (Figure 1a). Deviation of approximate value  $\hat{G} = \hat{G}_{appr}$  from exact value  $\hat{G} = \hat{G}_{exact}$  {calculated with due regard for all terms in relation (149) and equation (151)} is shown in Figure 2b. Exact solution at  $N = \hat{N}$  is not so difficult to find, because at this point  $\mu_n^a = 0$ . It is clear from Figure 2b that agreement  $\hat{G}_{appr}$  with  $\hat{G}_{exact}$  is quite good.

From expression (154), it follows that

$$\hat{G} = \left\{ 1 - 2 \times \frac{\hat{L}_{ef}}{W} \times \operatorname{th} \left( \frac{W}{2\hat{L}_{ef}} \right) \right\} \times (\mu_n \times \hat{\tau}_n + \mu_p \times \hat{\tau}_p) \times \frac{E_0}{W}, \quad (159)$$

where

$$\hat{L}_{ef} = \sqrt{\hat{L}_a^2 + \hat{L}_E^2} \quad (160)$$

is effective ambipolar length (156) at  $N = \hat{N}$  (i.e., at  $\delta \cong \sqrt{A+B}$ ) and  $\hat{\tau}_n$  and  $\hat{\tau}_p$  are relevant electron (48) and hole (38) lifetimes. We can write that

$$\hat{\tau}_n = \frac{\sqrt{A+B}}{2 \times w_n \times N_D}, \quad \hat{\tau}_p = \frac{2 \times A + \theta \times B \times \sqrt{A+B}}{2 \times w_p \times N_D \times (A+B)}, \quad (161)$$

$$\hat{L}_a^2 = \frac{(A/\theta) + B \times \sqrt{A+B}}{(A+B) \times D_n + B \times D_p} \times \frac{D_n \times D_p}{w_n \times N_D}, \quad (162)$$

$$\hat{L}_E^2 = \frac{\varepsilon}{8 \times \pi \times q} \times \frac{2 \times A + (B + 1) \times \theta \times \sqrt{A + B}}{w_p \times N_D \times n_t \times \sqrt{A + B}} \times \frac{\mu_n \times \mu_p \times E_0^2}{(A + B) \times \mu_n + B \times \mu_p}. \quad (163)$$

Value  $\hat{L}_a$  is conventional ambipolar diffusion length at maximal extremum of function  $G(N)$  (Figure 1a) calculated in quasi-neutrality approximation, i.e., when parameter  $\xi$  is set to zero.

From expression (159), it follows that function  $\hat{G}$  depends non-monotonically on applied bias voltage  $V$  (Figure 2a). This is caused by increased  $\hat{L}_{ef}$  with increasing  $E_0 = V / W$  that provides progressive loss of photoexcited charge carriers resulting from increasing diffusive inflow of photocarriers to current contacts' electrodes with follow-up recombination.

Increase in effective ambipolar diffusion constant  $D_{ef} = D_n^a + D_E$  (coefficient before second derivative in equation (152)) causes elongation  $\hat{L}_{ef}$  with increasing  $E_0$ . In turn, photoinduced space charge  $\Delta\rho$  (4) causes monotonic increased  $D_{ef}$  with increasing  $E_0$ . The analysis of expressions (159)–(163) shows that function  $\hat{G}(V)$  reaches its maximum value (Figure 2c)

$$\hat{G}_{\max}(W) \cong \begin{cases} \frac{25}{72} \times \frac{W}{\hat{L}_a} \times \hat{G}_{\max}^{\max}, & \text{when } W < 2 \times \hat{L}_a, \\ \hat{G}_{\max}^{\max}, & \text{when } W > 2 \times \hat{L}_a \end{cases}, \quad (164)$$

when bias voltage is applied across sample  $V = V_{opt}(W)$ , where optimal bias voltage (Figure 2d)

$$V_{opt}(W) \cong \begin{cases} \tilde{E}_0 \times W, & \text{when } W < 2 \times \hat{L}_a \\ \frac{\tilde{E}_0 \times W^2}{2 \times \hat{L}_a}, & \text{when } W > 2 \times \hat{L}_a \end{cases} \quad (165)$$

Threshold value  $G = \hat{G}_{\max}^{\max}$  (for given physical parameters of semiconductor) and electric field intensity  $E_0 = \tilde{E}_0$  at which  $\hat{L}_{ef} = \sqrt{2} \times \hat{L}_a$ , are defined by the following expressions:

$$\hat{G}_{\max}^{\max} = (\mu_n \times \hat{\tau}_n + \mu_p \times \hat{\tau}_p) \times \frac{3 \times \tilde{E}_0}{25 \times \hat{L}_a}, \quad (166)$$

$$\tilde{E} = \sqrt{\frac{8\pi \times kT \times n_i}{\varepsilon}} \times \left(1 + \frac{A}{B}\right)^{1/4}. \quad (167)$$

## 8. Summary

Capture rate of excess charge carriers increases with increasing concentration  $N$  of deep impurity levels, i.e., recombination centers (traps). However, as shown in this chapter on the



example of single-level acceptor, this increase does not lead to unavoidable decrease in lifetime of excess electrons  $\tau_n$  and holes  $\tau_p$ , when nonequilibrium filling of recombination level states is very low. The matter is that lifetimes are determined not only by capture of excess charge carriers at equilibrium traps but also by bound-to-free transitions of electrons and holes from nonequilibrium capture centers due to thermal emission and by capture of equilibrium charge carriers at nonequilibrium traps as well. Therefore, lifetimes of excess charge carriers can be either more or less than the time of their capture at equilibrium traps and can be strongly non-monotonic functions of recombination center concentration (Figure 1b). In the case of acceptor recombination level, it can happen if recombination level is located in lower half of forbidden gap. In the case of donor recombination level, it must be located in upper half of forbidden gap. It is essential that the ratio of lifetimes in maximum and minimum of functions  $\tau_n(N)$  and  $\tau_p(N)$  can be several orders of magnitude (Figures 1b and 3b).

It seems, authors of article [25] have reported first about the availability of minimum and portion of weak growth (up to 24 %) on experimental dependence of excess charge carriers' lifetime on recombination center concentration, which increased because of bombarding sample by high-energy electrons. Many years later, increase in lifetime, presumably, caused by increasing  $N$ , but already gained in several times, was observed experimentally [32].

The main reason for giant splash of photoresponse in semiconductors with increasing recombination center concentration  $N$  (Figures 1a, 6, and 7) is the growth of charge carriers' lifetime in orders of magnitude.

This reason is also sufficient to provide increase, in order of magnitude and more, in efficiency of charge carriers' photoexcitation with increasing  $N$  (Figure 6).

At and about point  $N = \hat{N} \cong N_D$ , where charge carriers' lifetime is maximal, equilibrium concentration of charge carriers becomes small, where  $N_D$  is shallow dopant concentration. Therefore, increase in Demer photo-emf  $V_{ph}$  in several orders of magnitude (Figure 7) is caused by both strongly non-monotonic dependences  $\tau_n(N)$  and  $\tau_p(N)$  (Figure 1b) and highly non-monotonic dependence of sample dark resistance on concentration  $N$  [1-3, 8, 30, 31].

However, increase in orders of magnitude in charge carriers' lifetime with increasing  $N$  (Figure 1b) is not a good reason for the development of giant splash in photoelectric gain  $G$  with increasing  $N$  (Figure 1a). As follows from [18, 19],  $G$  increases with increasing charge carriers' lifetime, if ambipolar mobility  $\mu_a$  (see (74), (158), [2]) is equal to zero, or if there is no recombination on current contact electrodes ( $x=0$  and  $x=W$ ; see inset in Figure 1a).

In reality, recombination on contact electrodes occurs always to more or less extent [5, 9]. Therefore, under normal conditions ( $\mu_a \neq 0$ ), increase in lifetimes, beginning from some lifetime values, does not increase in photocurrent density  $I_{ph}$  [5, 18, 19].

Saturation in  $I_{ph}$  is most clear in the case of high-rate recombination at contact electrodes (sweep-out effect on contacts [5, 18, 19]), when there are no photocarriers at contacts, i.e., conditions (1) are fulfilled. At trap-assisted recombination, function  $\mu(N)$ , under the same conditions (27) and (41), when there are non-monotonic dependences  $\tau_n(N)$  and  $\tau_p(N)$ ,



vanishes at the same, up to small, correction value  $N \cong \hat{N}$ , at which functions  $\tau_n(N)$  and  $\tau_p(N)$  reach their maximal extrema  $\hat{\tau}_n$  and  $\hat{\tau}_p$  (Figure 2b and 2c). Therefore,  $I_{ph}$  and, consequently,  $G$  increase to the extent of increasing  $\hat{\tau}_n$  and  $\hat{\tau}_p$ . These are physical grounds of giant splash in photoelectric gain  $G$  with increasing  $N$  (Figure 2a). Above mentioned results of strict analytical calculations (i.e., outside commonly used local approximation of quasi-neutrality) show that photoinduced local space charge affects substantially on giant splash of semiconductor photoelectric response with increasing concentration of recombination centers.

Strict solutions of problems concerning the quantity of photoexcited electrons  $N_{ph}$  and holes  $P_{ph}$  and Demer photo-emf  $V_{ph}$  may be fundamentally different from solutions obtained in approximation of quasi-neutrality  $\tilde{N}_{ph}$ ,  $\tilde{P}_{ph}$ , and  $\tilde{V}_{ph}$ , respectively.

It may be that  $P_{ph} / N_{ph} > 1$  even if hole lifetime  $\tau_p$  is much less than electron lifetime  $\tau_n$  (Figure 2b). At the same time, in approximation of quasi-neutrality,  $\tilde{P}_{ph} / \tilde{N}_{ph} = \tau_p / \tau_n < 1$  (Figure 2b). At point  $N = \hat{N}$ , at which functions  $N_{ph}(N)$ ,  $P_{ph}(N)$ , and  $V_{ph}(N)$  reach maximum values, and for thin samples (with thickness along light propagation  $\bar{W} < 0.1$  cm), in surroundings of point  $N = \hat{N}$ , solutions obtained in quasi-neutrality approximation may differ from solution outside quasi-neutrality in several orders of magnitude (Figures 8–11). Moreover, even  $\bar{W} \rightarrow \infty$ , neglecting by photoinduced space charge is not always possible, i.e., it is not always possible to solve problem in quasi-neutrality approximation. The reason is that when recombination level is deep enough, then shielding length of photoexcited space charge may be of the order of ambipolar diffusion length of charge carriers.

At sweep-out effect on contact electrodes, splash of  $G(N)$  with increasing  $N$  depends non-monotonically on applied voltage  $V$  across the sample (Figure 2a). That non-monotonic behavior is not related to heating of charge carriers or lattice and charge carriers injecting contacts. The reason is the increase in effective ambipolar diffusion constant  $D$  (coefficient before second derivative in equation (81) determining the distribution of photocarriers) with increasing  $V$ , leading to huge loss of photocarriers due to faster diffusion to contacts and subsequent recombination. In turn, increase in  $D \sim V^2$  is caused by photoinduced local space charge. What is important is that, at optimum voltage  $V_{op}$  (Figure 2d), value  $G$  can have several orders of magnitude (Figure 2c) at high concentrations of recombination centers.

As shown in [33], when recombination impurity  $N$  has three charged states (two-level recombination center), then, again, strong increase in  $\tau_n(N)$  and  $\tau_p(N)$  with increasing  $N$  may occur; moreover dependences  $\tau_n(N)$  and  $\tau_p(N)$  may have two charged states and two minimums and maximums. Opposite to single-level recombination center, in the case of two-level recombination center, maximum  $G(N)$ , as shown in [34], can be reached at lower concentrations  $N$  and have greater peak value. Photoelectric gain  $G$ , to the left of maximum value  $G(N)$ , is larger in the case of two-level recombination center, than in single level. The reason is the low ambipolar mobility of charge carriers in the case of two-level recombination center.

As shown in [35], significant growth of charge carriers' lifetimes with increase in concentration of recombination impurity in certain range could be stimulated by uncontrolled (background) doping by deep impurities. Even two maximums can occur.

Above-mentioned regularities occur at arbitrarily low-level photoexcitation and they become the more evident, the wider the semiconductor bandgap.

The theory of giant splash of photoresponse in semiconductors at low-level illumination with increasing concentration of recombination centers could develop further through the generalization of boundary conditions on semiconductor surfaces and current contact electrodes, accounting for nonuniformity of charge carriers' photoexcitation along the line of current flow and fluctuation processes. The study of nonstationary (frequency domain and transient) characteristics is of particular interest.

From physical essence of considered effects, it follows that similar effects can occur in other mediums with recombination of dissociative or ion-ion type, for example, in multicomponent plasma [36]. More details about topic are given in [37-46].

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## References

- [1] Milns A. Deep Impurities in Semiconductors. New York: John Wiley and Sons; 1973.
- [2] Smith R. Semiconductors. London: Cambridge University Press; 1978.
- [3] Sze S. Physics of Semiconductor Devices. New York: John Wiley and Sons; 1981.
- [4] Bube R. Photoelectronic Properties of Semiconductors. Cambridge: Cambridge University Press; 1992.

- [5] Rogalski A., editor. Infrared Photon Detectors. Bellingham: SPIE Press; 1995.
- [6] Colinge J.-P. & Colinge C. Physics of Semiconductor Devices. New York: Springer; 2002.
- [7] Lutz G. Semiconductor Radiation Detectors. Berlin: Springer; 2007.
- [8] Sze S. & Ng Kwok K. Physics of Semiconductors Devices. New York: John Wiley and Sons; 2007.
- [9] Rogalski A. Infrared Detectors. New York: CRC Press, Taylor & Francis Group; 2011.
- [10] Hall R. Recombination of electrons and holes in germanium. Phys. Rev. 1952; 87(2) 387.
- [11] Shockley W. & Read W. Statistics of the recombination of holes and electrons. Phys. Rev. 1952; 87(5) 835-842.
- [12] Blackmore J. Semiconductor Statistics. Oxford: Pergamon Press; 1962.
- [13] Ryvkin S. Photoelectric Effects in Semiconductors. New York: Consultants Bureau; 1964.
- [14] Blackmore J. Solid State Physics. London: Cambridge University Press; 1985.
- [15] Downey P.; Martin R.; Nahory R. & Lorimor O. High speed, ion bombarded InGaAs photoconductors. Appl. Phys. Lett. 1985; 46(4) 396.
- [16] Schaelin A.; Loepfe R; Melchior H.; Suter M. & Woelfli W. Beryllium-bombarded  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$  and InP photoconductors with response times below 3 ps. Mater. Sci. Eng. B 1989; 2(1-3) 81-85.
- [17] Balberg I. & Naidis R. Sensitization of the minority carriers lifetime in a photoconductor. Phys. Rev. B 1998; 57(12) R6783-6787.
- [18] Rittner E. Electron processes in photoconductors. Photoconductivity Conference at Atlantic City, November 1954, R.G. Breckenbridge, B. Russel & E. Hahn (eds.), 215-268, New York: Wiley & Sons; 1956.
- [19] Shacham-Diamand J. & Kidron I. Contact and bulk effects in intrinsic photoconductive infrared detectors. Infrared Phys. 1981; 21(2) 105-115.
- [20] Drugova A. & Kholodnov V. Concept of the realization of the ultimate possible photogain in intrinsic photoresistors under given concentration of the impurity recombination centres. Proceedings of International Semiconductor Device Research Symposium, v. 1, 197-200, December 1995, Charlottesville, Virginia, USA.
- [21] Keyes R., editor. Optical and infrared detectors. Berlin: Springer; 1977.
- [22] Beneking H. Gain and bandwidth of fast near-infrared photodetectors: A comparison of diodes, phototransistors, and photoconductive devices. IEEE Trans. Electron. Dev. 1982; ED-29(9) 1420-1430.

- [23] Kholodnov V. Giant burst of photoconductivity in semiconductors upon an increase in the concentration of recombination centers. JETP Lett. 1998; 67(9) 685-691.
- [24] Osipov V. & Kholodnov V. Theory of the diodes with radiative and unradiative impurity recombination. Sov. Phys. Semicond. 1970; 4(12) 1932-1937.
- [25] Wertheim G. Transient recombination of excess carriers in semiconductors. Phys. Rev. 1958; 109(4) 1086-1091.
- [26] Tsang W., editor. Lightwave Communication Technology: Photodetectors. Semiconductors and Semimetals v. 22, Part D, Willardson R. & Beer A. (eds.), New York: Academic Press; 1985.
- [27] Kireev P. Physics of Semiconductors (in Russian). Moscow: Higher School; 1969.
- [28] Moss T. Optical Properties of Semiconductors. London: Butterworth; 1959.
- [29] Pikus G. Foundations of the Semiconductor Devices Theory (in Russian). Moscow: Nauka; 1965.
- [30] Auth J.; Genzow D. & Herrmann K. Photoelektrische Erscheinungen (in German). Berlin: Akademie-Verlag 1977.
- [31] Bonch-Bruevich V. & Kalashnikov S. Semiconductor Physics (in Russian). Moscow: Nauka; 1990.
- [32] Basheleishvili Z.; Garnyk V.; Gorin S. & Pagaeva T. Change of the minority carrier lifetime under irradiation of high-ohmic p-type silicon. Sov. Phys. Semicond. 1984; 18(9) 1074-1075.
- [33] Kholodnov V. & Serebrennikov P. Critical importance of the single-level approximation to account for the highly non-monotonic dependences of carrier lifetimes on recombination impurity concentration. Tech. Phys. Lett. 1997; 23(4) 268-270.
- [34] Kholodnov V. & Serebrennikov P. Effect of photo-induced space charge on dependence of intrinsic threshold photoresistor amplification on two-level recombination impurity concentration. Proceedings SPIE, v. 5126, 352-356, Bellingham: SPIE Press; 2003.
- [35] Kholodnov V. & Serebrennikov P. Dependence of electron and hole lifetimes on the majority recombination impurity concentration in the presence of auxiliary carrier trapping centers. Tech. Phys. Lett. 1997; 23(12) 969-971.
- [36] Rayzer Yu. Physics of Gas Discharge (in Russian). Moscow: Intellect; 2009.
- [37] Drugova A. & Kholodnov V. Character of the dependences of non-equilibrium electron and hole lifetimes on the concentration of recombination centers in impurity-type recombination semiconductors. Solid-St. Electron. 1995; 38(6) 1247-1252.
- [38] Kholodnov V. Contribution to the Hall-Shockley-Read theory of recombination. Semiconductors 1996; 30(6) 538-544.

- [39] Kholodnov V. Giant splash of photoelectric amplification in photoconductors with increasing of recombination centers' concentration under weak optical radiation from the region of intrinsic absorption (in Russian). XV International Scientific and Engineering Conference on Photoelectronics and Night Vision Devices, Abstracts, p. 14, October 1998, Moscow, Russia.
- [40] Kholodnov V. Super non-monotonic dependence of gain in threshold intrinsic photoresistors on the concentration of recombination centers (in Russian). International Conference "Applied Optics-98", Abstracts, p. 88, December 1998, Sankt-Petersburg, Russia.
- [41] Kholodnov V. Gigantic splash of the weak optical radiation gain in intrinsic threshold photoconductive devices (photoresistors) upon an increase in the concentration of recombination centers. Proceedings of SPIE v. 3819, 98-115, Bellingham: SPIE Press; 1999.
- [42] Kholodnov V. & Drugova A. On impact of the photo-induced space charge upon semiconductor photoresponse dependence on the concentration of recombination centers under weak optical radiation. Proceedings of SPIE v. 4340, 186-212, Bellingham: SPIE Press; 2000.
- [43] Kholodnov V. & Drugova A. The influence of a photoinduced volume charge on the weak-radiation-induced interband photoproduction of carriers in semiconductors with impurity recombination centers. Tech. Phys. Lett. 2002; 28(9) 714-718.
- [44] Kholodnov V. & Drugova A. The principal role of non-quasi-neutrality of semiconductor photoexcited plasma in effect a giant splash the photoresponse of homogeneous semiconductors with increasing of recombination centers concentration (in Russian). VI Russian Conference on Semiconductor Physics, Abstracts, p. 204-205, October 2003, Sankt-Petersburg, Russia.
- [45] Kholodnov V. & Drugova A. On the degree to which the increase in the concentration of the recombination centers raises the efficiency of inter-band photoexcitation of carriers under weak optical radiation. Proceedings of SPIE v. 5126, 367-380, Bellingham: SPIE Press; 2003.
- [46] Kholodnov V. & Drugova A. The degree of efficiency enhancement of interband photoexcitation of the electromotive force of Dember at the cost of increasing the recombination centers concentration at weak optical radiation. XIX International Scientific and Engineering Conference on Photoelectronics and Night Vision Devices. Abstracts, p. 190, May 2006, Moscow, Russia.